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A hybrid heuristic for the uncapacitated single allocation hub location problem

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Abstract

The uncapacitated single allocation hub location problem (USAHLP), with the hub-and-spoke network structure, is a decision problem in regard to the number of hubs and location–allocation. In a pure hub-and-spoke network, all hubs, which act as switching points for internodal flows, are interconnected and none of the non-hubs (i.e., spokes) are directly connected. The key factors for designing a successful hub-and-spoke network are to determine the optimal number of hubs, to properly locate hubs, and to allocate the non-hubs to the hubs. In this paper two approaches to determine the upper bound for the number of hubs along with a hybrid heuristic based on the simulated annealing method, tabu list, and improvement procedures are proposed to resolve the USAHLP. Computational experiences indicate that by applying the derived upper bound for the number of hubs the proposed heuristic is capable of obtaining optimal solutions for all small-scaled problems very efficiently. Computational results also demonstrate that the proposed hybrid heuristic outperforms a genetic algorithm and a simulated annealing method in solving USAHLP.

Keywords: Hub-and-spoke; Simulated annealing; Tabu list; Uncapacitated hub location problem

1. Introduction

Hub-and-spoke networks consolidate traffic flows from different origins and ship them via hubs to different destinations so as to reduce the overall transportation cost. In pure hub-and-spoke networks, all hubs, which act as switching points for internodal flows, are interconnected and none of the non-hubs (i.e., spokes) are directly connected (i.e., all flow must be routed through the hubs). Each of the other non-hub nodes is allocated to a single hub (single allocation) or multiple hubs (multiple allocation) and is referred to as a spoke. Many studies have indicated that the implementation of hub-and-spoke networks has improved the operation of the distribution function. Typical applications of hub-and-spoke networks have arisen in passenger airlines [1–3], express package delivery firms [4], message delivery networks [5], trucking industry [6,7], telecommunication system [8], supply chain of chain stores such as Wal-Mart [9], and many other areas.

There are several types of hub-and-spoke network problems [10]. For instance, the well studied p-hub median problem (p-HMP). In the p-HMP, the number of hubs p is determined a priori. The objective is to locate the hubs and to allocate the non-hubs to the hubs so that the total transportation cost is minimized. The transportation cost includes three components: the collection cost incurred during the transportation from the origin to its allocated hub; the transfer cost incurred during the transportation between hubs; and the distribution cost incurred during the transportation from the allocated hub to the destination. It is assumed that the

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transportation costs on all inter-hub links are discounted by a factor α to reflect a scale merit on the inter-hub links.

In the uncapacitated hub location problem (UHLP) the number of hubs is not predetermined, as in the p-HMP. The number of hubs is a decision variable and a fixed cost for establishing a hub is included in the objective function. Although the UHLP is more commonly encountered in practice, yet it attracts less attention than the p-HMP. In this paper two approaches to determine the upper bound for the number of hubs along with a hybrid heuristic based on the simulated annealing (SA) method, tabu list from the tabu search (TS), and improvement procedures are proposed to resolve the UHLP with single allocation (USAHLP). Computational characteristics of the proposed approach are evaluated through extensive computational experiments using two benchmarking test instances: the Civil Aeronautics Board (CAB) and Australia Post (AP) data sets. Computational performance of the proposed approach was compared with the optimal/best solutions found in the literature.

The rest of this paper is divided into five sections. The previous related research is reviewed in Section 2. A mathematical model for the USAHLP is given in Section 3. The approaches to determine the upper bound for the number of hubs and the proposed hybrid heuristic are detailed in Section 4. Computational results are reported in Section 5. Conclusions and suggestions for future research are discussed in the last section.

2. Previous related research

Since the 1980s, hub-and-spoke network problems have attracted a lot of research interest. Campbell [11] presented a mixed $\frac{0}{1}$ integer linear formulation for the p-HMP with multiple allocation (UMApHMP). Ernst and Krishnamoorthy [12] presented a heuristic based on shortest paths and obtained exact solutions using enumeration and branch-and-bound methods. For the single allocation p-HMP (USApHMP), O'Kelly [13] showed that the problem is NP-hard and developed two enumeration-based heuristics. In the first heuristic, each non-hub is allocated to its nearest hub. In the second heuristic, each non-hub is allocated to its nearest or second-nearest hub. A TS heuristic and a greedy randomized adaptive search heuristic were proposed by Klincewicz [14]. Skorin-Kapov and Skorin-Kapov [15] developed a TS heuristic, in which locating the p hubs and allocating the spokes to the hubs are made simultaneously. Ernst and Krishnamoorthy [16] presented a new linear programming (LP) formulation for the problem and proposed an SA heuristic to obtain upper bounds for the LP problem. They then applied a branch-and-bound method to obtain the exact solutions. Campbell [17] showed that the solution for the multiple allocation p-HMP provides a lower bound for the single allocation p-HMP and developed two greedy-exchange heuristics. A hybrid heuristic, based on SA and TS, was proposed by Chen and Wu [18].

The UHLP has also been the subject of considerable research. Unlike the p-HMP, the number of hubs is a decision variable and a fixed cost for establishing a hub is included in the objective function. O'Kelly [19] formulated the USAHLP as a quadratic $\frac{0}{1}$ optimization problem and proposed a heuristic solution approach to deal with it. A mixed $\frac{0}{1}$ LP formulation for the USAHLP was presented by Campbell [11]. Abdinnour-Helm and Venkataramanan [20] suggested a branch-and-bound and a genetic algorithm (GA) to solve the USAHLP. Aykin [2] proposed a branch-and-bound algorithm and an SA based greedy-interchange heuristic to resolve the USAHLP. A hybrid heuristic, based on the GA and TS was presented by Abdinnour-Helm [21]. Topcuoglu et al. [22] proposed a GA-based procedure and an SA heuristic to solve the USAHLP. The results obtained by Topcuoglu et al. match the best solutions found in the literature.

For the UHLP with multiple allocation (UMAHLP), Klincewicz [23] proposed an algorithm, based on dual ascent and dual adjustment techniques within a branch-and-bound scheme, to obtain the exact solution. An improved version of Klincewicz's dual ascent approach was developed by Mayer and Wagner [24]. Boland et al. [25] developed preprocessing procedures and tightening constraints for the mixed $\frac{0}{1}$ LP formulation to obtain exact solutions. For different solution approaches to the different classes of hub-and-spoke network problems, interested readers may refer to Aykin [26], O'Kelly and Miller [27], Bryan [28], Bryan and O'Kelly [29], and Campbell et al. [10].

It is well known that the USAHLP is NP-hard [13]. When dealing with a large instance encountered in real world situations, in the worst case, it may not be possible to obtain an optimal solution in a reasonable time. Since SA and TS have been applied to solve a number of combinatorial problems with fairly good results, in this paper two approaches to determine the upper bound for the number of hubs along with a hybrid heuristic based on the SA method, tabu list from the TS, and improvement procedures are proposed to resolve the USAHLP to find optimal/near optimal solutions in a more efficient manner. Computational characteristics of the proposed approach are evaluated through extensive computational experiments using the widely tested CAB and AP data sets.

3. Model

A mathematical model for the USAHLP is given in this section to describe the problem structures. First, underlying assumptions are listed below:

(1) there are n nodes in the network;
(2) the number of hubs is a decision variable and there is a fixed cost associated with establishing a hub;
(3) all hubs are interconnected and none of the non-hub nodes are directly connected (i.e., all flow must be routed through the hubs);