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Linear logic as a tool for planning under temporal uncertainty

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ABSTRACT

The typical AI problem is that of making a plan of the actions to be performed by a controller so that it could get into a set of *final* situations, if it started with a certain *initial* situation.

The plans, and related winning strategies, happen to be finite in the case of a finite number of states and a finite number of *instant* actions.

The situation becomes much more complex when we deal with planning under *temporal uncertainty* caused by actions with *delayed effects*.

Here we introduce a tree-based formalism to express plans, or winning strategies, in finite state systems in which actions may have *quantitatively delayed effects*. Since the delays are non-deterministic and continuous, we need an infinite branching to display all possible delays. Nevertheless, under reasonable assumptions, we show that infinite winning strategies which may arise in this context can be captured by finite plans.

The above planning problem is specified in logical terms within a Horn fragment of affine logic. Among other things, the advantage of linear logic approach is that we can easily capture 'preemptive/anticipative' plans (in which a new action β may be taken at some moment within the running time of an action α being carried out, in order to be prepared before completion of action α).

In this paper we propose a comprehensive and adequate logical model of strong planning under temporal uncertainty which addresses infinity concerns. In particular, we establish a direct correspondence between linear logic proofs and plans, or winning strategies, for the actions with quantitative delayed effects.

1. Introduction and motivating examples

Linear logic has been shown to be an adequate tool for sorting out planning problems in deterministic as well as in non-deterministic domains [19,20,15].

The main advantage of linear logic approach is a direct and transparent correspondence between proofs for Horn linear logic sequents and plans for AI planning problems. In many cases this allows us to decrease significantly the combinatorial costs associated with searching large spaces [15,16].

The complexity results of [15,16] rely upon the assumption that the actions in question cause only *instant* effects, so that is the duration of the actions equals zero.

In this paper we address the planning problems under *temporal uncertainty* about the effects of actions [2,10] where the time duration does matter. Adding such a 'real time' direction makes the planning problem much more complicated. In particular, plans become winning strategies, and the planning objective is to find a plan that is guaranteed to achieve the goal even within the "worst-case scenario".

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The aim of the paper is to provide a strict correspondence between proofs and plans even within this temporal setting.

We will illustrate peculiarities and subtleties of the problem with the following simplified version of an example from [10]:

Example 1.1. Assume that a ship is scheduled to leave its original seaport (call it `‘there’`) to be serviced at `‘here’`. The move takes two to five days.

The ship can be serviced either on a normal dock (then she will stay docked two to three days), or on an express dock (then she will stay docked at most one day). But the express dock should be reserved two days in advance, and must be taken exactly two days after the moment the reservation has been made.

The question is to make a plan of actions to *guarantee* that, under any circumstances, the ship will be serviced within seven days? □

The positive answer to Example 1.1 is given, for instance, with the following plan:

l_1: At the initial moment 0, let the ship be bound for `‘here’`. Go to l_2.
l_2: If the ship comes in `‘here’` at some moment t_2 less than 4 time units, go to l_3.
Otherwise, go to l_4 (“If Plan A fails, go to Plan B”).
l_3: At this moment t_2, put her in the normal dock to be serviced. Go to l'_3.
l'_3: Having serviced the ship by some moment t'_2, stop.
(In total, it takes at most $t'_2 \leq (t_2+3) \leq 7$ days)
l_4: At moment t_1 such that $t_1 = 4$, make a reservation for the express dock. Go to l_5.
l_5: When the ship eventually comes in `‘here’` at some t_2, go to l_6.
l_6: At moment t_3 such that $t_3 = t_1+2$, put her in the express dock to be serviced. Go to l'_6.
l'_6: Having serviced the ship by some moment t'_3, stop.
(In total, it takes at most $t'_3 \leq (t_3+1) \leq (t_1+3) \leq 7$ days)

(1)

Remark 1.1. Solving planning problems, we have to address the following issues:

(a) “The guaranteed success, not simple reachability/compatibility”

Following the recommendations of the above plan (1), one can never be punished, since the plan represents a *winning strategy* that envisages *all* possible delays on the road from the initial situation to a final one.

In particular, at every point, the plan provides all preconditions for the corresponding action to be enabled at the given point.

On each of the execution branches, its timestamps form a non-decreasing sequence, with providing compatibility of the time constraints along the branch.

(b) “Preemptive/anticipative actions are vital”

In our example, line l_4 recommends to choose some moment t_1 within the waiting time for the ship’s move from `‘there’` to `‘here’` and to make a reservation for the express dock *in advance* before the ship’s move has been actually completed.

Moreover, we can show that any winning solution to Example 1.1 must include such a ‘preemptive/anticipative’ action: in the case of delays around 6 time units we would have failed if we had allowed the reservation action only after the above *move* action had been fully completed.

(c) “The lock-unlock discipline”

For each action α, the pairs of events “*start an action* α” and “*the action* α *is completed*” form in time a sequence of non-overlapping pairs.

In addition to that, the above plan is *perfect* from the garbage collection point: however the termination step we get, each of the actions involved has been already completed.

2. Real time

We are dealing with the following mathematical model.

A global continuous measurable quantity `time` is assumed in which events occur in irreversible succession from the past through the present to the future.

The *time advance* will be specified with the following `‘Tick’` axioms:

$$\mathtt{T}(t) \vdash \mathtt{T}(t+\varepsilon) \tag{2}$$

where $\mathtt{T}(t)$ denotes *“Time is t”*, and ε is an arbitrary positive real.

Time delays are generally qualified in terms of time intervals such as: “*It takes two to five days.*” Therefore, we will invoke the following basic facts related to time intervals.

As atomic formulas we consider $(t' \leq t+h)$, and $(t' < t+h)$, and $(t' = t+h)$, etc. where t and t' are *time variables*, measured in time units, and h is a *real constant*, measured in time units. These atomic formulas may be combined by ‘product’ $\otimes$ and ‘disjunction’ $\oplus$.