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On the descriptional power of heads, counters, and pebbles

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Abstract

We investigate the descriptional complexity of deterministic two-way k-head finite automata (k-DHA). It is shown that between non-deterministic pushdown automata and any k-DHA, $k \geqslant 2$, there are savings in the size of description which cannot be bounded by any recursive function. The same is true for the other end of the hierarchy. Such non-recursive trade-offs are also shown between any k-DHA, $k \geqslant 1$, and $\mathsf{DSPACE}(\log) = $ multi-DHA. We also address the particular case of unary languages. In general, it is possible that non-recursive trade-offs for arbitrary languages reduce to recursive trade-offs for unary languages. Here we present huge lower bounds for the unary trade-offs between non-deterministic finite automata and any k-DHA, $k \geqslant 2$. Furthermore, several known simulation results imply the presented trade-offs for other descriptional systems, e.g., deterministic two-way finite automata with k pebbles or with k linearly bounded counters.

Keywords: Descriptional complexity; State complexity; Non-recursive trade-offs; Finite automata

1. Introduction

Formal languages can have many representations in the world of automata, grammars and other rewriting systems, language equations, logical formulas etc. So it is natural to investigate the succinctness of their representation by different models. The regular languages are one of the first and most intensely studied language families. It is well known that non-deterministic finite automata (NFA) can offer exponential savings in size compared

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with deterministic finite automata (DFA). Concerning the number of states, 2^n is a tight bound for the NFA to DFA conversion [13]. Further asymptotic bounds are $O(n^n)$ for the two-way DFA to one-way DFA conversion [13], $2^{O(n^2)}$ for the two-way NFA to one-way DFA conversion [23], $O(\sqrt{2^n})$ for the two-way DFA to one-way NFA conversion [1], and $O(2^{3n})$ for the two-way NFA to one-way NFA conversion [1]. The latter reference is a valuable source for further simulation results.

All trade-offs mentioned with respect to the number of states are bounded by recursive functions. But, for example, there is no recursive function which bounds the savings in descriptional complexity between deterministic and unambiguous pushdown automata [30]. In [25] it is proved that the trade-off between unambiguous and non-deterministic pushdown automata is also non-recursive. Recent results involving the parallel model of cellular automata can be found in [10]. In particular, non-recursive trade-offs are shown between DFA and real-time one-way cellular automata (real-time OCA), between pushdown automata and real-time OCA, and between real-time OCA and real-time two-way cellular automata.

A comprehensive survey of descriptional complexity of machines with limited resources in [3], which is a valuable source for further results and references.

Nevertheless, some challenging problems of finite automata are open. An important example is the question of how many states are sufficient and necessary to simulate two-way NFA with two-way DFA. The problem has been raised in [23] and partially solved in [7,9,28].

When certain problems are difficult to resolve in general, a natural question concerns simpler versions. To this regard, promising research has been done for unary languages. It turned out that this particular case is essentially different from the general case. The problem of evaluating the costs of unary automata simulations has been raised in [28]. In [2] it has been shown that the unary NFA to DFA conversion takes $e^{\Theta(\sqrt{n \ln(n)})}$ states, the NFA to two-way DFA conversion has been solved with a bound of $O(n^2)$ states, and the costs of unary two-way to one-way DFA conversion reduces to $e^{\Theta(\sqrt{n \ln(n)})}$. Several more results can be found in [11,12]. Furthermore, in [10] it is shown for real-time OCA that non-recursive trade-offs for arbitrary languages reduce to recursive trade-offs for unary languages.

Here we investigate the descriptional complexity of deterministic two-way k-head finite automata (k-DHA). In particular, we consider the trade-offs between non-deterministic pushdown automata and k-DHA, for any $k \geqslant 2$, and the trade-offs between any k-DHA, $k \geqslant 1$, and the deterministic log-space bounded Turing machines, whose languages are exactly the languages accepted by the union of all k-DHA. All these trade-offs are shown to be non-recursive. For unary languages it is not known whether the trade-offs are recursive or not. Here we present huge lower bounds between non-deterministic finite automata and any k-DHA, $k \geqslant 2$. Furthermore, these lower bounds increase with the number of heads in a nice way. Provided minimality can be shown, these bounds can also serve as lower bounds between k-DHA and $(k+1)$-DHA.

In the next section, we define the basic notions and present a preliminary example. Section 3 is devoted to the study of the mentioned non-recursive trade-offs. Unary languages and the huge lower bounds are considered in Section 4. Finally, in Section 5 the results are adapted to other types of acceptors, e.g., deterministic two-way finite automata with k pebbles or with k linearly bounded counters. Some concerned and related open questions are discussed.