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Efficient time-optimal feedrate planning under dynamic constraints for a high-order CNC servo system[☆]

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HIGHLIGHTS

- A relationship between the tracking error and the input signal is established.
- The tracking error constraint for a CNC system is reduced to a kinematic constraint.
- The nonlinear constraints on jerk are reduced to linear constraints.
- Feedrate planning under confined tracking error is reduced to convex optimization.

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ABSTRACT

In this paper, the time-optimal feedrate planning problem under confined feedrate, axis velocity, axis acceleration, axis jerk, and axis tracking error for a high-order CNC servo system is studied. The problem is useful in that the full ability of the CNC machine is used to enhance the machining productivity while keeping the machining precision under a given level. However, the problem is computationally challenging. The main contribution of this paper is to approximate the problem nicely by a finite-state convex optimization problem which can be solved efficiently. The method consists of two key ingredients. First, a relationship between the tracking error and the input signal in a high-order CNC servo system is established. As a consequence, the tracking error constraint is reduced to a constraint on the kinematic quantities. Second, a novel method is introduced to relax the nonlinear constraints on kinematic quantities to linear ones. Experimental results are used to validate the proposed method.
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1. Introduction

The problem of time-optimal feedrate planning along a given parametric tool path has received a significant amount of attention in the CNC machining literature due to its ability to increase the productivity of CNC machining by using the full ability of the machines [1–13]. The feedrate planning problem is usually formulated as a time-minimum optimal control problem under kinematic constraints such as confined feedrate, axis acceleration, jerk, and even jounce, and efficient algorithms have been proposed to solve the problem. The acceleration bounds are introduced to reduce inertia and prevent mechanical shocks. The jerk and jounce bounds are used to generate smooth feedrate profiles aimed at improving the machining quality.

For various reasons, such as the inertia of the CNC axes and inaccurate modeling of the CNC dynamic system, the tracking error is not guaranteed to reach the desired level even if the acceleration and jerk are bounded. A common method to reduce tracking errors is to use a closed-loop controller which calculates the difference between the desired signal and the feedback signal in real time and generates a control signal to minimize the dynamic error. Many algorithms along this line have been developed, such as the cross-coupled control strategy [14,15], the model-referenced adaptive control [16], the predictive control [17], and the learning control [18]. In order to use these closed-loop methods, users need to access the control system, which demands more from the end-users.

An alternative approach is to combine "open-loop" feedrate planning with dynamic precision control by adding a tracking error bound as a new constraint in the feedrate planning phase. An advantage of this approach is that accessing the control system is not required, and hence this is more convenient for the end-users. Dong and Stori [19,20] considered the dynamic error information in the feedrate planning phase by approximating the tracking error

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with the linear part of its Taylor expansion. Ernesto and Farouki [4] solved the problem of compensating for inertia and damping of the machine axes by a priori modifying the commanded tool path. In [21,22], Lin, Tsai, et al. used critical point approaches to generate a feedrate with confined contouring errors. In [23], a linear programming method is proposed to solve the feedrate planning problem under confined tracking error for CNC systems based on PD controllers.

In this paper, the time-minimum feedrate planning problem under confined feedrate, axis velocity, axis acceleration, axis jerk, and axis tracking error is studied. To be practical, dynamic systems with PID controllers are considered, where the tracking error satisfies a third-order differential equation. The time-minimum feedrate planning problem in this situation is strongly nonlinear, and there exist no efficient algorithms for solving it at present. The main contribution of this paper is to reduce the time-minimum feedrate planning problem into a finite-state convex optimization problem whose global optimal solution can be computed efficiently. The work consists of two key ingredients. First, a relationship between the tracking error and the input signal in high-order CNC servo systems is established. As a consequence, the tracking error constraint can be reduced to a constraint on a linear combination of kinematic quantities such as accelerations and jerks. Second, a novel method is introduced to relax the nonlinear constraints involving the jerk to linear constraints. Experimental results show that the new convex optimization problem gives a nice approximation to the original problem and can be solved efficiently.

In contrast to the work reported in [19,4,23], high-order CNC servo systems are considered, which allows the usage of PID controllers, while the work reported in [19,4,23] only considered second-order systems for P or PD controllers. Furthermore, the relaxation of the tracking error in this paper is theoretically guaranteed to be valid, while the one given in [19] is an approximation. Also, our approach reduces the optimal feedrate planning problem into a convex programming problem which can be solved efficiently. The method proposed in [10], although more general than ours, is less efficient. In contrast to [21,22], our approach generates an approximate time minimum feedrate, while the feedrate generated with methods in [21,22] is not time minimum. And in contrast to [9], which initiates the powerful convex optimization approach, we consider a much more general and non-convex problem which is reduced to a convex optimization problem, while the problem considered in [9] is itself convex.

The paper is organized as follows. Section 2 describes high-order dynamic models and shows how to simplify the tracking error constraint. Section 3 presents an efficient method to solve the time-minimum feedrate planning problem by converting it to a convex optimization problem. In Section 4, experimental results are used to demonstrate the effectiveness of the approach. In Section 5, concluding remarks are given.

2. Tracking error simplification for a high-order CNC servo system

In this section, we will show that the tracking error constraint can be replaced by a constraint on kinematic quantities such as accelerations and jerks.

2.1. Tracking error of a high-order CNC servo system

Suppose that the CNC machine is controlled by M axes; the subscript $\tau \in \{1, \ldots, M\}$ will represent these axes. Each axis is powered by a DC motor satisfying an nth-order linear system whose transfer function is [24, p. 70]

$$H_\tau(s) = \frac{x_\tau(s)}{X_\tau(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0},$$

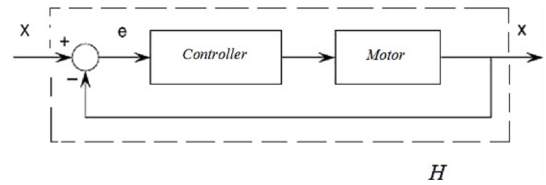

Fig. 1. DC servo system.

where X_τ and x_τ are the commanded axis location and the actual axis location, respectively.

Using the closed-form position control shown in Fig. 1, we can calculate the transfer function between the *tracking error* $e_\tau = X_\tau - x_\tau$ and the input signal X_τ:

$$\begin{aligned} G(s) &= \frac{e_\tau(s)}{X_\tau(s)} \\ &= \frac{(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0) - (b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0)}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \\ &= \frac{c_l s^l + c_{l-1}s^{l-1} + \cdots + c_1 s + c_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \end{aligned}$$

where $l \le \max(m, n)$. Using the inverse Laplace transformation of the above equation, if the initial values of $e_\tau(t)$ and its derivatives are zero, then the tracking error $e_\tau(t)$ satisfies the following differential equation:

$$\begin{aligned} &a_n \frac{\mathrm{d}^n e_\tau}{\mathrm{d}t^n} + a_{n-1}\frac{\mathrm{d}^{n-1} e_\tau}{\mathrm{d}t^{n-1}} + \cdots + a_1 \frac{\mathrm{d}e_\tau}{\mathrm{d}t} + a_0 e_\tau \\ &= c_l \frac{\mathrm{d}^l X_\tau}{\mathrm{d}t^l} + c_{l-1}\frac{\mathrm{d}^{l-1} X_\tau}{\mathrm{d}t^{l-1}} + \cdots + c_1 \frac{\mathrm{d}X_\tau}{\mathrm{d}t} + c_0 X_\tau, \end{aligned} \tag{2.1}$$

where t represents time. In order for the above system in e_τ to be stable, the real parts of the eigenvalues of the linear system on the left-hand side of (2.1) are assumed to be negative [24, p. 76]. More precisely, the roots of the following character equation of system (2.1)

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0 = 0 \tag{2.2}$$

are assumed to have negative real parts.

Now, the problem of optimal trajectory planning along a given tool-path can be formulated as the following time-minimum control problem under kinematic constraints and tracking error constraints:

$$\begin{aligned} &\min T \quad s.t. \left|\frac{\mathrm{d}X_\tau}{\mathrm{d}t}\right| \le X^1_{\max}, \left|\frac{\mathrm{d}^2 X_\tau}{\mathrm{d}t^2}\right| \le X^2_{\max}, \ldots, \\ &\left|\frac{\mathrm{d}^m X_\tau}{\mathrm{d}t^m}\right| \le X^m_{\max}, |e_\tau| \le E_{\max}, \end{aligned} \tag{2.3}$$

where $\tau \in \{1, \ldots, M\}$ represents the axes, m is a positive integer to be given by the user, $X^j_{\max}$ are positive real numbers, and $e_\tau(t)$ satisfies Eq. (2.1).

Note that the number m determines the smoothness of the velocity functions to be obtained. For instance, if $m = 2$, then the acceleration is bounded and the velocity function is continuous; if $m = 3$, then the acceleration and the jerk are bounded, and the velocity function is differentiable; and if $m = 4$, then the acceleration, jerk, and jounce are bounded, and the acceleration function is differentiable. Generally speaking, the smoother the velocity function is, the less vibration occurs, as shown in [5].

2.2. Simplification of the tracking error constraint

The last constraint of (2.3) is difficult to deal with since it will lead to complicated equations when solving the optimization problem via discrete methods. In this section, we will show that

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