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A triangulation-based hole patching method using differential evolution

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HIGHLIGHTS

- A new hole patching method is proposed to repair the defective model.
- The information on both sides of the boundary around the considered hole is used.
- The points in the hole region are predicted by differential evolution.
- The operations of mesh optimization are used to improve the quality of the mesh.

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ABSTRACT

In this work, a new hole patching method (namely as, HPDE) is proposed to repair the damaged or ill-scanned three dimensional objects in real engineering applications. Our method differentiates from other related algorithms mainly on the following three aspects. Firstly, our algorithm sufficiently utilizes the point information around the considered hole for each prediction by constructing point correspondences on both sides of the boundary curve of the hole; secondly, the missing points in the hole region are predicted by the algorithm of differential evolution (DE), which is used to obtain the topological and geometrical structures of the mesh in the hole region; thirdly, operations of mesh optimization are adopted for improving the quality of the obtained triangulation mesh. Numerical results on kinds of holes with complex shape and large curvature, and a comparison with two recently proposed algorithms verify the effectiveness of the algorithm, further experiments on the noisy data points illustrate the robustness of the algorithm against noise.

1. Introduction

Methods for constructing a mathematical model from a given set of three dimensional (3D) points have been applied into a wide variety of fields, such as engineering design, virtual reality, movie making and data visualization. Surface reconstruction has drawn much attention from last decades of years and a lot of methods are reported [1–4]. It is more often that the provided points are represented with piecewise triangles (namely as triangulation model), such as the Marching Cubes method in [5], the crust algorithm in [6], direct advancing front method and modified decimation method in [7], and a divide and conquer Delaunay triangulation in [8].

However, the constructed model may not be directly applicable for the real application. In one case, the provided triangulation model is damaged or the points in the considered model are ill-scanned. In another case, the provided 3D points can also be the measured parameter values from the simulation experiments, and it may come up that the data referring the parameter values in the abnormal conditions is difficult to be obtained by the simulations, which may be the result from the difficulty of these experiments or the high cost and time-consuming of the simulation. For these two cases, there still exist concerned regions (hole regions) where the point information is unknown and needs to be predicted based on the constructed model, thus, the hole patching method is resorted to.

In this work, a new hole patching method (HPDE) based on defective triangulation model is devised to patch challenging holes with large curvature. Based on the boundary points and boundary directional vectors, the proposed HPDE utilizes algorithms of differential evolution (DE) and constrained triangulation sequently to obtain the topological and geometrical structures of the triangulation mesh in the hole region, which is further resorted to the algorithms of smoothing and mesh optimization to improve the mesh quality. The proposed HPDE differs from other hole patching methods mainly on the following aspects. (1) The proposed HPDE combines the frequently used algorithms of DE, constrained triangulation and mesh optimization in the application of hole patching.

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(2) Different from other algorithms obtaining the topological structure in advance, our method obtains the topological and geometrical structures of the mesh in the hole region at the same time. (3) HPDE sufficiently utilizes the directional vectors at each two corresponding base points on the boundary curve around the considered hole for the prediction, which is different from other algorithms either using only one side of boundary information or not using the directional vectors at the boundary points.

This paper is organized as follows. Some related work of hole patching are introduced in Section 2. Section 3 presents the description of the classical DE algorithm. The proposed HPDE is described in Section 4. Section 5 presents all the numerical results on some kinds of test models. Finally, some discussions and conclusions are given in Sections 6 and 7, respectively.

2. Related work

Method of hole patching is to fill in a considered region by expanding the constructed model around the region. For the hole patching, the surrounding region of the hole region is filled with the point information of the constructed model, then hole regions are patched with the methods which are similar to interpolation by incorporating this information. A few methods referring to the hole patching have been sprang up in the past few years. Based on the representation of the constructed model, these methods are summarized into mainly two categories, one is volume-based, the other is surface-based.

Generally, the volume-based methods patch the holes by first assigning signs to a set of voxels with a signed distance function, then the point information in hole region is completed in the volume representation, which is used to approximately obtain the missing interface of the geometry by some surface-extracting methods such as the marching cubes method. The model with holes was converted to the volumetric representation, on which form the model was naturally repaired by parity count method or ray stabbing method, then the repaired model was converted back to a waterproof polygonal model [9]. The surface model was first converted to the model with volume representation, then a diffusion was employed to extend the surface to form a watertight model [10]. This method can deal with geometrically and topologically complex holes. A novel hole-filling algorithm was proposed for volumetric objects where not only the outer surface of 3D objects but also the solid volumetric objects were closed filled [11]. The main disadvantage of these volume-based methods is that they may miss some important features of the original model when converting to and from a volume, although these methods can often produce a watertight model in a robust way.

The surface-based methods deal with the data points directly and they often utilize the information in a local region around the considered hole. For these surface-based methods, the topological structure of the triangulation mesh in the hole region is first obtained, then the geometrical structure is obtained by some other algorithms. The defects and holes were proposed to be repaired by a 3D triangulation method which is to minimize the total sum of edge lengths by dynamic programming [12]. The constrained triangulation of the unfolded 3D points on a plane was embedded into the triangular mesh by minimizing an energy surface [13]. The radial basis function interpolator using neighboring edges was employed to fill the holes [14]. The topological structure of the prediction points in the hole region was obtained by the advancing front method, and the geometric structure of these points was determined by solving the Poisson equation [15]. The surface-based approach and a two-step volume-based method-heat diffusion and the Poisson surface reconstruction were incorporated to repair holes with geometric and topological complexities [16]. It was proposed that complex polygonal holes were filled in a piecewise manner so as to obtain the entire hole triangulation with the piecewise planar triangulation [17]. The patching mesh in hole region was obtained by iteratively refining and smoothing the preciously optimized mesh according to the lengths of the adjacent triangles, based on an initial triangulation mesh of the boundary points of the considered hole region [18,19]. A grey prediction model was proposed to predict and adjust the coordinates of the newly added points by obtaining two controlled variables—the normal vector and the included angle size [20], this method tends to maintain the variation trend of the boundary points by utilizing the point information adjacent to the prediction points. While the used point information is only on one side of the boundary, the variation trend from one side to another side in the hole region cannot be detected, which may result in losing fidelity for holes where the points vary uniformly from one side to the other. The hole regions were patched with non-uniform rational B-spline surface by utilizing several layers of adjacent triangles [21], this method first obtains several interpolation spline curves using some adjacent points around the hole region, then some discrete points in the hole region are extracted from these interpolation curves, which are used for a B-spline surface interpolation to obtain the geometrical structure of the triangulation mesh. Because of the characteristic of the B-spline function that the shape taken by the spline is with minimum elastic energy, the patched surface in the hole region is apt to be flat which may also result in losing fidelity.

These reported methods adopt their own strategies to predict the points in the hole region, and achieve competitive results on their concerned test models. However, how to sufficiently utilize the information around the considered hole and how to effectively handle challenging holes are still hot topics in this field.

3. DE algorithm

As a stochastic algorithm which is first proposed in [22,23], DE and its variants are frequently adopted in various kinds of real engineering applications [24]. Different from classical gradient-based optimization algorithms, DE adopts one greedy strategy to proceed to global optimum from multiple positions of the searching region based on a random generated population. Without utilizing the gradient information of the objective function, DE is often the real alternative for a variety of non-differentiable and non-convex problems which are difficult for gradient-based algorithms.

Without loss of generality, assume the following minimization problem is considered:

$$\min_{\mathbf{x}} f(\mathbf{x}) \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$, and $x_j \in [l_j, u_j]$, $j = 1, \ldots, n$. For stochastic algorithms which are population based, denote population as $\mathbf{X} \triangleq \{\mathbf{X}_1, \ldots, \mathbf{X}_N\}$, where N is the number of individuals. Denote $\mathbf{X}^g \triangleq \{\mathbf{X}_1^g, \ldots, \mathbf{X}_N^g\}$ as the population at the generation of g.

DE algorithm employs the diversity information implied in the population to generate the mutation individual, the most frequently used version of DE algorithm is *DE/rand/1/bin*, where *rand* denotes choosing random vectors for mutation, 1 denotes employing one difference term for the mutation procedure, and *bin* denotes generating a trial individual by accepting the parameter values from the mutation individual one at a time. This version is detailed described in Algorithm 1 and our proposed algorithm is devised based on this version.

There are other mutation and crossover strategies which compose other versions of DE [24]. In addition to *rand*/1 mutation strategy, *best*/1 and *rand*/2 are also frequently used, which are listed as follows:

1. *best*/1:

$$\mathbf{V}_i = \mathbf{X}_*^g + F \cdot (\mathbf{X}_{r_2^{(i)}}^g - \mathbf{X}_{r_3^{(i)}}^g) \tag{6}$$