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Freeform surface flattening based on fitting a woven mesh model

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Abstract

This paper presents a robust and efficient surface flattening approach based on fitting a woven-like mesh model on a 3D freeform surface. The fitting algorithm is based on *tendon node mapping* (TNM) and *diagonal node mapping* (DNM), where TNM determines the position of a new node on the surface along the warp or weft direction and DNM locates a node along the diagonal direction. During the 3D fitting process, strain energy of the woven model is released by a diffusion process that minimizes the deformation between the resultant 2D pattern and the given surface. Nodes mapping and movement in the proposed approach are based on the discrete geodesic curve generation algorithm, so no parametric surface or pre-parameterization is required. After fitting the woven model onto the given surface, a continuous planar coordinate mapping is established between the 3D surface and its counterpart in the plane, based on the idea of geodesic interpolation of the mappings of the nodes in the woven model. The proposed approach accommodates surfaces with darts, which are commonly utilized in clothing industry to reduce the stretch of surface forming and flattening. Both isotropic and anisotropic materials are supported.

Keywords: Surface flattening; Woven material; Strain energy; 3D fitting; Freeform surface

1. Introduction

Surface flattening is an important process in many applications (e.g. aircraft industry, ship industry, shoe industry, apparel industry, etc.). In the traditional process of footwear industry, the profile of the shoe upper layer is first estimated and then cut out; after sewing together the pieces of the layer, a foot shape mould is inserted to deform the leather to a desired shape [1]. In the aircraft industry, structures reinforced by woven fabrics are commonly used [2]. Similar to the footwear case, profiles of the woven fabrics are estimated and cut out, and then they are laid onto a certain 3D shape. In both cases, the profile of the material is still conjectured in practice by human based on trial-and-error and this estimation is quite time consuming and inaccurate. In the Computer-Aided Design (CAD) of products, people expect to obtain an accurate profile. Actually, they want to obtain the profile in a reverse way: first designing the 3D surface of the product on a CAD system, and then determine the corresponding 2D profile of the surface. This is exactly the following surface flattening problem:

Problem Definition Given a 3D freeform surface and the material properties, find its counterpart pattern in the plane and a mapping relationship between the two so that, when the 2D pattern is folded into the 3D surface, the amount of distortion—wrinkles and stretches—is minimized.

In this paper, we present a surface flattening technique based on fitting a woven-like mesh (*woven mesh*) model onto a 3D surface M. Two mapping methods: *tendon node mapping* (TNM) and *diagonal node mapping* (DNM) are proposed to initially locate the nodes of a woven mesh on the given surface. In the tendon node mapping, two mutually perpendicular geodesic curves are generated on M which are called *tendon*s since they will not be moved in the ensuing energy releasing process and they are mapped into two perpendicular straight lines on the planar woven before the fitting. The tendon nodes are located on the tendon curves with equal distance. The diagonal node mapping method is then incorporated to position new nodes based on the other three located nodes belonging to the same

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quad in the woven mesh. Thus, by a propagation procedure, the nodes can be fitted on M one by one. During the fitting of nodes, strain energies at the fitted nodes are released by a diffusion process. The strain energy is defined based on the geodesic distance of adjacent nodes and their Euclidean distance on the surface. The difference between the original 2D woven mesh and the given surface is minimized, so the deformation between the 2D profile and the 3D freeform surface is minimized. Both the node mapping and movement in our approach are based on the discrete geodesic curve generation algorithm [4]; therefore, different from other existing methods [5–9], no parametric surface or pre-parameterization is required by us. After fitting a woven mesh model, a planar coordinate mapping is developed to compute the 2D coordinate of every point on M. The proposed fitting technique accommodates surfaces with darts which are commonly adopted in practice to reduce the distortion of surface forming and flattening. Also, for the strain energy minimization, not only isotropic but also anisotropic materials can be simulated.

The freeform 3D surface considered in this paper is represented as a two-manifold polygonal mesh with a boundary, which is topologically equivalent to a disk. The mesh is a complex of vertices and the connectivity between the vertices—here we adopt the data structure in [3] to store the mesh. Using this data structure, we can easily obtain the adjacent relationship of vertex–vertex, vertex–edge, vertex–face, and edge–face.

The paper is organized as follows. We will first review some related work in surface flattening. The woven mesh model is then introduced. The detail fitting methodology is presented in Section 4, in the sequence of tendon node mapping, diagonal node mapping, boundary propagation, and strain energy minimization. Section 5 describes the planar coordinate mapping scheme which establishes the continuous mapping relationship between every point on a given surface and its flattened 2D counterpart. A number of experimental examples are then presented to illustrate the proposed flattening algorithm, and comparisons are made with two other known surface flattening algorithms (one is pure geometry-oriented and another is energy-based). Finally in Section 7 we summarize the paper.

2. Related work

Due to its importance, in both theory and practice, research in surface flattening has been active for a number of years, and not limited to only design and manufacturing. In the following we give a short summary on the various related developments over the past few decades.

2.1. Parameterization

The flattening of a triangular 3D mesh, which provides a bijective mapping between the mesh and a triangulation of a planar polygon, plays an important role in parameterization and texture mapping. An excellent survey of recent advances in mesh parameterization is given in [10], see also the references therein. Floater [11] investigated a graph-theory based parameterization for tessellated surfaces for the purpose of smooth surface fitting; his parameterization (actually a planar triangulation) is the solution of linear systems based on convex combination. In [12], Hormann and Greiner used Floater's algorithm as a starting point for a highly non-linear local optimization algorithm which computes the positions for both interior and boundary nodes based on local shape preservation criteria. The method is promising, but it is not clear if the procedure is guaranteed to converge to a valid solution. A quasi-conformal parameterization method based on a least-squares approximation of the Cauchy–Riemann equations is introduced in [13], where the defined objective function minimizes angle deformation. Desbrun et al. [14] developed an efficient parameterization algorithm minimizing the distortion of different intrinsic measures of the original mesh. However, in both [13] and [14], the linear stretch is not considered. Sheffer and de Sturler [15,16] presented a texture mapping algorithm that causes small mapping distortion. Their algorithm consists of two steps: (1) using the Angle Based Flattening (ABF) parameterization method to provide a continuous (no foldovers) mapping, which concentrates on minimizing the angular distortion of the mapping and hence unavoidably often leads to relatively large linear distortion; (2) to reduce the linear distortion, an inverse mapping from the plane to the result of ABF is computed to improve the parameterization—the improved result has low length distortion. In [17], a texture stretch metric is introduced to minimize the linear distortion via non-linear optimization. Since non-linear numerical optimization is conducted in [15–17], these approaches are time consuming. Most recently, in [18], a fast and simple method for generating a low-stretch mesh parameterization is developed. It starts from any other parameterization (e.g. the Floater shape preserving parameterization [11], or the intrinsic parameterization [14]) and then improves the parameterization gradually by a diffusion process using the stretch metric of [17]. It can significantly improve the stretch in a mesh parameterization. However, since the boundary vertices are not moved, the 2D boundary profile depends on the initial parameterization. When the natural boundaries are required (as mentioned earlier in our problem definition), they use the intrinsic parameterization [14]. Since in [14] the stretch is not minimized, the resultant 2D profiles are seldom satisfied in either its length or the enclosed area.

2.2. Strain-energy minimization

McCartney et al. [19] flatten a triangulated surface by minimizing the strain energy in the 2D pattern. The 3D surface is first triangulated using Delaunay triangulation.