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Tolerance envelopes of planar mechanical parts with parametric tolerances[☆]

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Abstract

We present a framework for the systematic study of parametric variation in planar mechanical parts and for efficiently computing approximations of their tolerance envelopes. Part features are specified by explicit functions defining their position and shape as a function of parameters whose nominal values vary along tolerance intervals. Their tolerance envelopes model perfect form least and most material conditions (LMC/MMC). Tolerance envelopes are useful in many design tasks such as quantifying functional errors, identifying unexpected part collisions, and determining device assemblability. We derive geometric properties of the tolerance envelopes and describe four efficient algorithms for computing first-order linear approximations with successive accuracy. The results from experiments on 14 realistic part models demonstrate that on average, our algorithms are an order of magnitude faster and more accurate than the commonly used Monte Carlo simulation, and produce better results than the computationally expensive Taguchi method.

Keywords: Tolerancing; Parametric models; Tolerance zones; Mechanical design

1. Introduction

Manufacturing and assembly processes are inherently imprecise, producing parts that vary in size and form. The need to control the quality of the production and to manufacture parts interchangeably led to the development of tolerance specifications. Tolerance specifications are the critical link between the designer and the manufacturer. Designers prefer tight tolerances to ensure that the part will fit in the assembly and perform its function. Manufacturers, on the other hand, prefer loose tolerances to lower the production cost and decrease the need for quality machine tools and precision measurement machines. Tolerance analysis methods play a key role in bridging between the two.

Tolerance allocation is difficult even to the most skilled of designers because it requires identifying the critical interactions of toleranced dimensions, which often have complex dependencies. Tolerancing methods have been developed and incorporated into most modern CAD software. Given a tolerance allocation, tolerance analysis consists of predicting the effect of the allowed variations on the design functions. Tolerance synthesis consists of finding tolerance intervals that meet the functional requirements at the lowest cost.

A key problem in tolerance analysis is computing the tolerance envelope of a part from its tolerance specification. Tolerance specifications define a family of parts consisting of all valid instances of the part. The tolerance zone of a part is the difference between the smallest volume containing all part instances and the largest volume contained in all part instances. Its boundaries, called the part tolerance envelope, define the worst-case variability of the part features, and thus model perfect form most and least material conditions (MMC/LMC). Part tolerance zones are useful in design tasks such as quantifying

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functional errors, identifying unexpected part collisions, and determining device assemblability.

Recent research in computer-aided tolerancing (CAT) describes methods for defining and computing tolerance zones for individual features from their tolerance specifications [1,5]. However, many issues regarding tolerance zones for entire parts remain open: what is their geometric complexity, what are good approximations, and how to efficiently compute them. Previous works are limited by the descriptive power of their variational models, by the quality of the approximations they produce, and by their computational efficiency.

In this paper, we present a framework for the systematic study of parametric variation in planar mechanical parts and for efficiently computing approximations of their tolerance envelopes. The framework reflects current tolerancing practice, incorporates common tolerancing assumptions, and exposes the computational trade-offs. Of the two commonly used tolerance specification methods [25], geometric and parametric, we chose parametric specification because it is best suited for functional tolerancing, can be used to describe most geometric specifications, and has a simple, mathematically well-defined semantics within which part variability can be studied analytically. In the proposed model, part features are specified by explicit functions defining their position and shape as a function of parameters whose nominal values vary along tolerance intervals. We derive geometric properties of the worst-case tolerance envelopes and describe four efficient algorithms for computing first-order linear approximations with successive accuracy. We test the algorithms on 14 realistic part models and compare their quality and efficiency with the commonly used Monte Carlo and Taguchi sampling methods. The experiments demonstrate that on average, our algorithms are an order of magnitude faster and more accurate than Monte Carlo simulation, and produce better results than Taguchi's method, whose performance degrades exponentially with the number of parameters, and thus is inappropriate for interactive tolerance analysis.

The rest of the paper is organized as follows. In Section 2 we review previous methods of worst case tolerance analysis using tolerance zones and their counterparts in parametric spaces. In Section 3 we present the parametric tolerancing model and give a concrete example. In Section 4 we define the linear approximation of toleranced vertices and segments and derive results on the geometric properties and complexity of their tolerance envelopes. In Section 5 we describe four algorithms for the computation of segment tolerance envelopes, with successive complexity and accuracy. In Section 6 we present our experimental results, and in Section 7 we conclude with a summary and future work.

2. Previous work

Several models have been proposed to model parametric part variations These include simple-shaped regions around boundary points [11,26] and fixed-distance boundary offsets [15,18], which are computationally efficient but do not have the required semantics of the tolerance specification. Pottmann et al. [17] model freeform curves whose control points lie within convex tolerance zones. The tolerance zones traced by these curves are overly conservative because they ignore parameter dependencies which preclude significant regions of the zone.

Pino et al. [16] describe a kinematic model to simulate the 'motion' of the features tolerance zone but do not describe how to compute the entire part tolerance zone. Bhide et al. [3] use areal coordinates to describe the Tolerance Map, a convex volume of points corresponding to all possible locations and variations of a plane which can arise from geometrical tolerances on size, form, and orientation. Desrochers et al. [8] use a similar concept, in which the location and orientation of the plane are described by screw parameters of small displacement. Both these methods enable stack up analysis in an assembly, but do not compute the corresponding volumes for the entire part. Sacks and Joskowicz [19] describe a kinematic tolerance analysis method that computes contact tolerance zones of planar parametric parts in configuration space. The method uses a similar parametric part model and computes contact zones which complement the part tolerance envelopes used in this paper.

A few CAT packages provide tools for computing worst-case part tolerance zones [22]. Some compute tolerance zones from many randomly generated part shape instances drawn from a presupposed parameter distribution (the Monte Carlo method) [6], or sample instances of the part with extremal parameter values (the Taguchi method) [7]. These methods are expensive and incomplete, as mechanical assemblies typically have hundreds of features defined by tens of parameters. ADAPT [20], developed by Ford for internal use, computes the tolerance envelopes of parametric planar parts with procedural definitions [13]. It has one procedure for each of the many feature definition cases and incorporates ad-hoc simplifying assumptions that preclude quantifying the approximation error. The drawbacks of these methods motivate our work.

3. Tolerancing model

We propose the following tolerancing model for planar parts, which is very general in its semantics and has good computational properties.

Let A be a simple planar part whose boundary consists of curved segments. Its nominal shape and variation is defined by an m-dimensional parameter vector p. Each parameter has a nominal value and a tolerance interval, typically much