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Shape-preserving interpolation by fair discrete G^3 space curves

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Abstract

We present a new method for interpolation by a shape-preserving space curve with third-order geometric continuity. The curve is represented by a discrete sequence of vertices along with first, second, and third derivative vectors at each vertex, where derivatives are with respect to arc length. A user need only specify the number of vertices and a single tension factor in addition to the control points to be interpolated. The method consists of minimizing the total variation of curvature subject to the interpolation and shape-preservation constraints. The method is global but relatively efficient, and it consistently produces curves with pleasing graphical behavior.

Keywords: Fair curve; Interpolation; Shape-preserving; Space curve

1. Introduction

The problem of constructing visually pleasing or 'fair' curves is central to Computer Aided Geometric Design. Fairness is subjective and application dependent. It may be quantified in many ways, including minimization of functionals such as those discussed in (Roulier and Rando, 1994), and preservation of the shape properties of the data or control points. In addition to inducing fairness, shape constraints may be essential for reproducing data properties such as positivity, monotonicity, and convexity, especially in the case of planar curves. While the literature on shape-preserving planar curves goes back at least as far as Schweikert's introduction of the exponential tension spline (Schweikert, 1966), treatment of space curves is much more recent. Refer to (Asaturyan et al., 2001; Karavelas and Kaklis, 2000; Goodman et

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al., 1998) for the most recent work in this area. Greiner (1998) reviews the literature on the more general problem of optimization-based fairing.

Our method consists of minimizing the total variation of curvature (Moreton and Séquin, 1994) subject to the interpolation and shape-preservation constraints. This choice of fairness measure leads to a challenging computational problem, but results in very smooth and visually pleasing curves. In (Renka, 2004) we presented an effective method for minimizing variation of curvature subject to linear equality constraints in the form of user-specified tangents and curvature vectors (along with the control points). The shape-preservation constraints treated here are nonlinear inequalities which add considerably to the difficulty. We tried several constrained optimization methods (penalty, augmented Lagrangian, and projection methods) with limited success before getting excellent results with a logarithmic barrier method as described below.

As in (Renka, 2004), we represent the curve by a sequence of vertices, along with derivative vectors up to third order. This has two major advantages over the more conventional approach of using basis functions. It allows maximum flexibility with an arbitrarily large user-specified number of degrees of freedom in the fitting function, and it greatly simplifies the computational procedure: derivatives with respect to arc length involve simple divided differences rather than the complex parameter-dependent expressions that would otherwise be required, and integrals are accurately approximated by a simple quadrature rule.

Despite the simplifications noted above, an expression for the Hessian of the variation of curvature is extremely complicated, all but ruling out a second-order method. We therefore employ a first-order method, and we make this feasible by using a variable metric method based on Neuberger's theory of Sobolev gradients (Neuberger, 1997). This is the central idea of Renka (2004).

In Section 2 we define the problem in greater detail, Section 3 describes the method, test results are presented in Section 4, and Section 5 concludes the paper.

2. The problem

Given a sequence of control points $\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_n$ in $\mathbf{E}^3$, $n \geqslant 2$, we seek to construct a smooth (G^3 continuous) curve $\mathbf{f}$ that interpolates the control point data, minimizes the variation of curvature, and preserves the shape of the data in the sense that it reproduces the convexity and torsion of the control polygon. Note that, as described in Section 2.1 below, we use a discrete curve representation rather than a set of continuous basis functions, and thus the term 'control point' does not have its usual meaning. Also, the geometric continuity characterizes an underlying function (the minimizer of the variation of curvature) for which we construct a discrete second-order finite difference approximation. Furthermore, as discussed in Section 2.3, smoothness may be reduced by the constraints.

2.1. Curve representation

The free parameters defining a curve $\mathbf{f}$ consist of a sequence of $m+1$ vertices $\mathbf{f}_i$, $i = 0, \ldots, m$. The curve is represented by these vertices along with derivative vectors defined below. We define an integer array of indexes **index** such that $\mathbf{index}(0) = 0$, $\mathbf{index}(j) - \mathbf{index}(j-1) \geqslant 3$ for $j = 1, \ldots, n$, and $\mathbf{index}(n) = m$. Then the control point interpolation conditions are $\mathbf{f}_i = \mathbf{p}_j$ for $i = \mathbf{index}(j)$ and $j = 0, \ldots, n$. This may be thought of as a parameterization in which the knots (parameter values associ-