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Simple local interpolation of surfaces using normal vectors

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Abstract

A simple algorithm for surface interpolation is proposed. Its central idea is quadratic interpolation of a curved segment from the position and normal vectors at the end points, with the aid of generalized inverses. It is then used to recover the curvature of triangular or quadrilateral patches. The methodology has the following distinctive features: (i) The algorithm is efficient and completely local, requiring only the position vectors and normals given at the nodes of a patch, and hence it is suitable for parallel processing. (ii) The C^0 continuity is always attained, and errors in the normals diminish rapidly with the increase in the number of nodes. (iii) Since the approach can account for discontinuity (multiplicity) of normals, sharp edges and singular points, as well as non-manifolds, can be treated quite easily. (iv) Because of the low degree of the interpolation, it is rather robust and amenable to numerical analyses in comparison with the traditional cubic and more elaborate approximations. Validity and effectiveness of the formulation are checked through several examples.
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Keywords: Local interpolation; Normal vector; Generalized inverse; Quadratic polynomial patch; Sharp edge; Singular point; Non-manifold; Parallel processing

1. Introduction

Sophisticated surface interpolations through NURBS and Bézier parameterizations, etc. are powerful tools for CAD, where high level of continuity using a small number of patches is preferred. However, their

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application to numerical simulation represented by the finite element method (FEM) is rather limited, mainly due to the following reasons.

(A) Discretized models are already divided into a large number of small patches, for which elaborate interpolation is, in general, not necessary. In many engineering and applied physics, it is therefore more crucial to lower the complexity of interpolation.
(B) Complex surface expressions discourage rapid evaluation of quantities necessary for physical analyses, such as distance, cross-sectional area, volume and surface integrals. Hence many numerical simulation programs accept only *linear or quadratic* surface interpolators. For instance, modeling of contact mechanics involves solution for intersection of a patch and a line. The process is analytic for the quadratic parametric representation, since it leads to a quartic equation. However, closed solutions cannot be obtained if the interpolant is cubic or higher, and hence sophisticated geometric descriptions are generally impractical. Similar situations are very common in real-world problems; they usually impose severe constraints on the mathematical models.
(C) In contrast to the industrial design aiming at creation of fine shapes, simulation models are assumed to be already provided as CAD data, etc. Therefore, fidelity (convergence) to the original geometry is more important than visual quality, e.g., high level of smoothness. Here it should be stressed that *smoothness does not mean accuracy* of the algorithm. For example, Nielson's minimum norm network gives C^1 interpolation, but its degree of algebraic precision is only one (Nielson, 1983).
(D) Smooth local interpolators involve free parameters or assumptions on the derivatives. Such redundancy and a priori choice may be good for generating varied surfaces with pleasing shapes, but they are drawbacks to scientific applications where the accuracy needs to be respected. When a surface is already given and it is to be approximated by the traditional methods, the free parameters assuring the convergence are required to be evaluated. This is usually impossible, because the analytic properties of the original surface are generally unknown.
(E) Highly continuous interpolation results in equations coupled tightly across the patches. This can be a critical hurdle against fast computation through distributed processing, etc.
(F) Singular points, sharp edges and non-manifolds are often encountered in realistic applications. Most interpolation schemes presently available cannot deal with such intricate features in an efficient and robust way.

Because of the above restrictions, most researchers on FEM, etc., still resort to polyhedral models, but they oversimplify the original system neglecting the curvature, yielding significant errors in the analyses. A natural solution to this problem is to account for the normals of the original surfaces evaluated at the mesh vertices. However, existing interpolators summarized below are not suited for this type of quantitative simulation purposes.

There is a vast body of literature on smooth local interpolations. Since the approaches are quite common in geometric design, they are reviewed first. Piper (1987) proved that the tangent plane continuity of adjacent Bézier patches is always attained if and only if their degrees are quartic or higher. The author proposed an interpolation scheme for the quartic case, but it does not uniquely determine a surface only from the given data since the algorithm involves free parameters to be provided a priori. Peters (1991) suggested the singular parameterization, which assumes that the first and mixed derivatives of patches enclosing a vertex vanish at the parametric origin. Although the technique enables C^1 interpolation, it may cause 'bulgy' surfaces demanding minimization of the variation of the cross-boundary derivatives. The

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