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Matrix representation for multi-degree reduction of Bézier curves

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Abstract

In this paper, we consider multi-degree reduction of Bézier curves with constraints of endpoints continuity with respect to L_2 norm. The control points of the degree reduced Bézier curve can be obtained as a product of the degree reduction matrix and the vector of original control points. We find an explicit form of the multi-degree reduction matrix for Bézier curve with constraints of endpoints continuity.

Keywords: Bézier curves; Jacobi polynomials; Degree reduction; Endpoints continuity; Matrix representation

1. Introduction

Given control points $\{p_i\}_{i=0}^n$, a degree n Bézier curve is defined by

$$p(t) = \sum_{i=0}^{n} p_i B_i^n(t), \quad t \in [0,1], \tag{1}$$

where $B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$ is the Bernstein polynomial of degree n. The problem of degree reduction is to find control points $\{q_i\}_{i=0}^m$ which define the approximate Bézier curve

$$q(t) = \sum_{i=0}^{m} q_i B_i^m(t), \quad t \in [0,1], \tag{2}$$

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of degree m $(m < n)$ such that a suitable distance function $d(p,q)$ is minimized. Degree reduction of parametric curves was first proposed as the inverse problem of degree elevation (Forrest, 1972; Farin, 1983).

In this paper we are interested in finding a degree reduction matrix Q satisfying

$$\mathbf{q} = Q\mathbf{p}, \tag{3}$$

where

$$\mathbf{p} = (p_0, p_1, \ldots, p_n)^{\mathrm{t}} \quad \text{and} \quad \mathbf{q} = (q_0, q_1, \ldots, q_m)^{\mathrm{t}} \tag{4}$$

denote the vectors of the control points of the Bézier curves $p(t)$ and $q(t)$, respectively. When $m = n-1$, Sunwoo and Lee (2004) proposed a unified matrix representation for degree reduction of Bézier curves. They found that the degree reduction matrices for well known methods can be represented in a unified form, namely,

$$\mathbf{q} = A_\lambda \mathbf{p} \tag{5}$$

for a suitable matrix A_λ which is a generalized inverse of the degree elevation matrix. For multi-degree reduction ($m < n-1$), Lee and Park (1997) found the degree reduction matrix with respect to L_2 norm without endpoints continuity. They show that $\mathbf{q} = (T^{\mathrm{t}}QT)^{-1}T^{\mathrm{t}}Q\mathbf{p}$ for some matrix Q and the degree elevation matrix T. Also Ahn et al. (2004) proved the constrained polynomial degree reduction in the L_2 norm is equivalent to best weighted Euclidean approximation of Bézier coefficients. They derived the control points $\mathbf{q}$ of the form $\mathbf{q} = (T^{\mathrm{t}}WT)^{-1}T^{\mathrm{t}}W\mathbf{p}$. Although Lee and Park (1997) and Ahn et al. (2004) proposed the matrix representation of the multi-degree reduction, their methods require the computation of matrix inverses.

Chen and Wang (2002) derived an explicit form of the least squares solution of multi-degree reduction of Bézier curves with high order endpoints continuity. But their method, called MDR by L_2, contains some recursive formulas and requires converting a polynomial in Jacobi basis to Bernstein basis. By solving a recursive formula explicitly and finding a basis transformation matrix, we represent the method of Chen and Wang (2002) in a matrix form. The purpose of this paper is to find a multi-degree reduction matrix $Q_{m\times n}^{(r,s)}$ such that

$$\mathbf{q} = Q_{m\times n}^{(r,s)}\mathbf{p}, \tag{6}$$

where $Q_{m\times n}^{(r,s)}$ is determined only by m, n and the orders (r,s) of endpoints continuity.

The organization of this paper is as follows: We introduce some basic results in Section 2. Some results about Jacobi polynomials are stated in Section 3. Also the basis transformation between Jacobi and Bernstein polynomials is given in Section 3. Derivation of the degree reduction matrix $Q_{m\times n}^{(r,s)}$ is presented in Section 4 with an example.

2. Preliminaries

In this paper we use the least squares (L_2) norm defined by

$$d_2(p,q) = \sqrt{\int_0^1 \left|p(t) - q(t)\right|^2 \mathrm{d}t}. \tag{7}$$