Computers & Graphics 37 (2013) 316–332

Contents lists available at SciVerse ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Special Section on Expressive Graphics

Hamiltonian cycle art: Surface covering wire sculptures and duotone surfaces

Ergun Akleman [a,*], Qing Xing [a], Pradeep Garigipati [a], Gabriel Taubin [b], Jianer Chen [a], Shiyu Hu [a]

[a] Texas A&M University, United States
[b] Brown University, United States

ARTICLE INFO

Article history:
Received 5 September 2012
Received in revised form
30 December 2012
Accepted 7 January 2013
Available online 9 February 2013

Keywords:
Shape modeling
Computer aided sculpting
Texture mapping
Computational aesthetics
Computational art

ABSTRACT

In this work, we present the concept of "Hamiltonian cycle art" that is based on the fact that any mesh surface can be converted to a single closed 3D curve. These curves are constructed by connecting the centers of every two neighboring triangles in the Hamiltonian triangle strips. We call these curves *surface covering* since they follow the shape of the mesh surface by meandering over it like a river. We show that these curves can be used to create wire sculptures and duotone (two-color painted) surfaces.

To obtain surface covering wire sculptures we have developed two methods to construct corresponding 3D wires from surface covering curves. The first method constructs equal diameter wires. The second method creates wires with varying diameter and can produce wires that densely cover the mesh surface.

For duotone surfaces, we have developed a method to obtain surface covering curves that can divide any given mesh surface into two regions that can be painted in two different colors. These curves serve as a boundary that define two visually interlocked regions in the surface. We have implemented this method by mapping appropriate textures to each face of the initial mesh. The resulting textured surfaces look aesthetically pleasing since they closely resemble planar TSP (traveling salesmen problem) art and Truchet-like curves.

1. Introduction and motivation

In this work, we introduce a simple approach that provides methods to create a variety of artworks. Our approach is based on converting any given mesh surface into a closed 3D curve that follows the shape of the given surface. Our work is based on Gabriel Taubin's work on constructing Hamiltonian triangle strips on quadrilateral meshes [1–3].

In graph theory, a Hamiltonian path is a path in an undirected graph that visits each vertex exactly once. A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian path that is a cycle. Note that not every graph has a Hamiltonian cycle. Hamiltonian triangle strips are defined in duals of triangular meshes. Taubin shows that it is always possible to construct a triangular mesh from any given quadrilateral mesh such that the dual of the triangular mesh has an Hamiltonian cycle. Moreover, he presented simple linear time and space constructive algorithms to construct these triangle strips. His algorithms are based on splitting each quadrilateral face along one of its two diagonals and linking the resulting triangles along the original mesh edges. With these algorithms every connected manifold quadrilateral mesh without boundary can be represented as a single Hamiltonian generalized triangle strip cycle.

Using Taubin's algorithms to construct a closed curve in 3D is straightforward. One can simply connect centers of triangles in the triangle strip to obtain a control polygon in 3D. The resulting control polygon can be turned into a smooth curve using a parametric curve such as B-spline as shown in Fig. 1 [2]. These curves can be used for creating artworks. Designers of these curves have significantly large number of aesthetic possibilities. There are two ways to control aesthetic possibilities for surface covering curves:

- *Designing mesh structures*: The shape of any given surface can be approximated by a wide variety of meshes. Therefore, designers, by choosing different meshes, can obtain aesthetically different curves. Examples that show the effect of the structure of the underlying mesh on a spherical shape are shown in Fig. 1. In this figure, the control meshes are obtained using a variety of subdivision schemes available in TopMod3D such as honeycomb and pentagonal subdivisions [4–6].

* Corresponding author. Tel.: +1 9797390094.
E-mail addresses: ergun.akleman@gmail.com, ergun@tamu.edu (E. Akleman).

0097-8493/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cag.2013.01.004

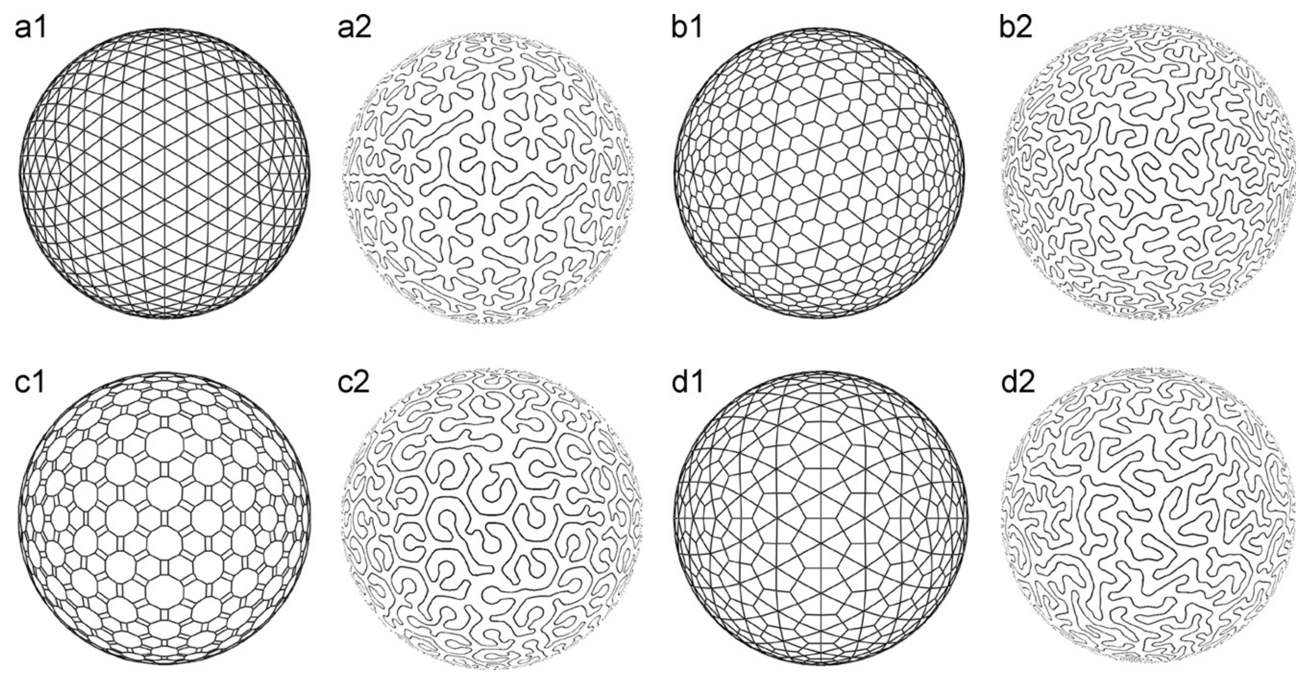

Fig. 1. Examples of surface covering curves on a sphere: spherical mesh surfaces are converted into closed 3D curves which follow the shapes of the original spheres. Back-faces in meshes and back-face parts of the curves are not drawn for cleaner images. (a1) Geodesic dome. (a2) Curve constructed from the mesh in (a1). (b1) A spherical mesh. (b2) Curve constructed from the mesh in (b1). (c1) A spherical mesh. (c2) Curve constructed from the mesh in (c1). (d1) A spherical mesh. (d2) Curve constructed from the mesh in (d1).

The spherical shapes are obtained by simply moving vertices of the meshes into a unit sphere. For the detailed discussion of how these structures can be obtained see [7].

- *Controlling shapes of curves*: Mathematically speaking, there are many ways to form surface covering curves for any given mesh [1,8]. This mathematical property provides additional aesthetic possibilities since designers can have additional control over the shapes of the curves. We prefer wavy curves like a meandering river since they resemble space filling curves [9] or TSP (traveling salesmen problem) art [10] embedded on surfaces.

1.1. Surface covering wire sculptures

To convert curves into 3D wires, we sweep a polygon or a line along the curve. This process normally requires rotation minimizing frames to avoid undesirable twists [11]. In our case since the curves are on surfaces it is possible to avoid twists by using surface normals to obtain Frenet frames (see [12] for details). Therefore, it is easy to convert these curves into 3D structures that can be shaded, rendered and even eventually 3D printed.

We have developed two methods to construct corresponding 3D ribbons and wires from given curves as extruded lines and polygons along the curves [2]. The first method, called constant-diameter, simply turns the curves into constant thickness ribbons or equal diameter wires. The second method, which we call variable-diameter, creates ribbons with varying thicknesses (or wires with changing diameters) that can densely cover the mesh surface. We have developed a system that converts polygonal meshes to surface filling curves, ribbons and wires. All the images of wire sculptures are direct screen captures from the system.

Fig. 2 shows an example obtained by using constant and variable diameter methods. Our variable diameter method guarantees that the sizes are relative to the underlying triangles. Therefore, the actual widths of ribbons are different in different parts of the mesh. Fig. 8 shows visual effects of constant vs. variable and ribbon vs. wire for the same spherical mesh.

1.2. Duotone surfaces

The Jordan curve theorem states that any simple closed curve in the plane separates the plane into two regions: the part that lies inside the curve and the part that lies outside it [13]. Although the theorem seems to be very intuitive, the proof is complicated since closed curves can be complicated sometimes such as fractal curves. Many artists observed this property to create artworks over plane by creating interesting curves such as fractal art, traveling salesmen problem (TSP) art and Truchet-like curves. Interestingly, Jordan's theorem is only correct for genus-0 surfaces. Any single curve on a surface with positive genus does not necessarily separate the surface into two regions.

To obtain duotone surface, we present a simple approach to construct surface covering curves that can separate surfaces into two regions [3] (see Figs. 1 and 3). Our method is based on a useful property of vertex insertion schemes such as Catmull–Clark subdivision: If such a vertex insertion scheme is applied to a mesh, the vertices of resulting quadrilateral mesh are always two colorable. Using this property, we can always classify vertices of meshes that are obtained by a vertex insertion scheme into two groups. We show that it is always possible to create a single curve that covers the whole surface such that all vertices in the first group are on one side of the curve while the other group of vertices are on the other side of the same curve.

We have implemented our approach using Truchet tiles where the boundary curve is not explicitly constructed but appears as the boundary of two regions formed by Truchet tiles. Therefore, our implementation can be considered as an embedding of duotone Truchet tiles over surfaces [14]. We therefore call our textured surfaces duotone surfaces. However, unlike duotone Truchet tiles our duotone surfaces guarantee only two regions separated by a single curve.