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Technical Section

Topological saliency ☆

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ABSTRACT

Topological methods have been successfully used to identify features in scalar fields and to measure their importance. In this paper, we define a notion of topological saliency that captures the relative importance of a topological feature with respect to other features in its local neighborhood. Features are identified by extreme points of an input scalar field, and their importance measured by the so-called topological persistence. Computing the topological saliency of all features for varying neighborhood sizes results in a saliency plot that serves as a summary of relative importance of all topological features. We develop a convenient tool for users to interactively select and inspect features using the saliency plot. We demonstrate the use of topological saliency together with the rich information encoded in the saliency plot in several applications, including key feature identification, scalar field simplification, and feature clustering.

1. Introduction

The use of topological methods is becoming popular for analyzing, visualizing, and exploring scalar fields. It is being used for a wide range of applications including topological simplification and cleaning [1–4], surface segmentation and parametrization [5–7], topology based shape matching [8,9], and designing transfer functions for volume rendering [10–13]. Distinguishing between significant and unimportant features of the input forms an integral part of the methodologies used in these applications. In this paper we focus on this theme to define a notion of saliency for topological features and explore its application to visual analysis of features.

1.1. Related work

Features in scalar fields are represented by critical points of the field. A common approach to identifying features in 3D geometric models is to first design a descriptor function that captures important information about the input and use critical points of such a descriptor function as representatives of features. Various methods have been proposed to produce a meaningful descriptor function. Once a descriptor function is given, one can measure the topological importance of the features (critical points) associated with it based on the so-called persistence homology, originally proposed by Edelsbrunner et al. [1]. Indeed, topological persistence has been demonstrated to be an effective importance measure and has been used for many applications, including scalar field simplification [14,15] and shape matching [8]. While the notion of persistence can effectively describe the importance of a feature with respect to an input scalar function or filtration, it is somewhat oblivious to other geometric information not encoded in the input function. In particular, it does not reflect how important a feature is relative to other features in its neighborhood.

In this paper, we aim to initiate a study in this direction by defining a notion of topological saliency for features in the input that captures the relative importance of a feature within a spatial neighborhood. We propose to do this following an approach similar to saliency models used in image and geometric mesh analysis. Many models for obtaining salient locations in images have been proposed [16–20]. In particular, Itti et al. [19] propose a model that computes the saliency of a pixel in an image based on the properties of pixels in its neighborhood. Lee et al. [21] extend this model to geometric features and propose a notion of mesh saliency that captures the saliency of a point in a surface or volume mesh. It is computed as the curvature at a point weighted by the average curvature within a small neighborhood.

Our topological saliency framework can be viewed as a way to combine geometry information with topological methods. Note

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that this measure is not intended to replace existing global importance measures. Rather, we expect it to complement existing measures when applied to the visual analysis of features.

We remark that the theme of combining geometry and topology is not new. For example, Carr et al. [22] employed geometric measures computed on contour trees to find and simplify less significant features. Weber et al. [12] used Reeb graphs to identify significant features in volumetric data in order to design transfer functions for rendering the volume. The concept of topological landscapes [23,24] provides an intuitive view of the data by displaying its topological features, abstracted by the contour tree, as a terrain. In Agarwal et al. [25], the authors aim to use a descriptor function to encode certain geometric information of interest, and use topological persistence to identify geometric features from this function. The concept of "localized homology" was later proposed by computing the homology from local pieces to global pieces, so that the generators of homology classes are localized in local pieces [26]. Reininghaus et al. [27] proposed an importance measure for critical points in two dimensional scalar fields called the scale space persistence, which combines the notion of deep structure of the scale space with topological persistence. The scale space persistence is computed by accumulating the persistence values of a critical point through its evolution in the scale space [28].

1.2. Motivating example

Consider the terrain shown in Fig. 1(a) with seven peaks. Existing topology-based methods would ignore peak F even though it dominates a large area of the domain in the sense that it remains an important feature within a large neighborhood size. Similarly, based on persistence, peaks B and D would have been declared as equally important even though D is surrounded by other peaks of similar height making it not as dominant as B. In general, spatial distribution of topological features has not been considered while measuring the size of a feature and its significance.

Such scenarios are common when studying medical data obtained using diffuse optimal tomography. For example, Fig. 1(b) shows a volume rendering of a breast dataset having a tumor. The high persistent features in the input correspond to the fibre bundles on the periphery of the volume, shown in yellow. However, the tumor corresponds to a low persistent feature that is isolated within the volume. Therefore, using persistence alone, it may not be possible to identify the tumor as a significant feature of the input.

1.3. Results

We address the problem raised in the above example by defining a notion of *topological saliency* that considers the presence or absence of other features within the neighborhood while measuring the importance of a topological feature. A feature in this paper is always represented by an extremum (minimum or maximum) of the input scalar field. The topological saliency of a feature is computed as a weighted average of the topological persistence of features within its local neighborhood. We also introduce a *saliency plot* that is generated by computing the topological saliency of all features for varying neighborhood sizes. This plot can be considered as an augmentation or refinement of persistence, obtained by injecting certain spatial geometry information into it. We also propose the use of topological saliency for simplifying the input in order to remove noise.

Further, we develop visualization software to facilitate the use of the saliency plot. In particular, the software can compute the saliency plots of minima and maxima for a given input field as well as a decomposition of the input domain around these features. It also allows the users to interactively select and inspect features of the input using the saliency plot. Finally, we demonstrate the use of this software and the concept of topological saliency in the following applications.

- Identify key features that may be missed by standard persistence. In particular, we use topological saliency to identify breast tumors.
- Identify craters on Mars. We use topological saliency in conjunction with standard persistence to identify significant craters on the surface of Mars.
- Extract similar features. We use the topological saliency plot to identify and group similar features of the input.

2. Background

We briefly introduce some necessary notations and refer the readers to appropriate textbooks [29–31] for more precise definitions and comprehensive discussions of these concepts.

2.1. Morse function

Let $\mathbb{M}$ denote a d-manifold with or without boundary. Given a smooth, real-valued function $f : \mathbb{M} \to \mathbb{R}$ defined on $\mathbb{M}$, the *critical points* of f are exactly where the gradient becomes zero. The

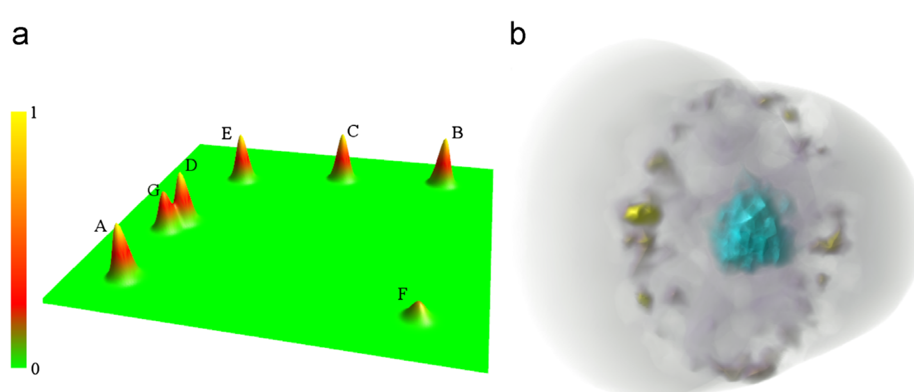

Fig. 1. (a) A sample terrain with seven peaks. Traditional topological methods identify peaks A, B, C, D and E as important. Even though peak F remains a lone peak for a significantly large part of the domain, it is not considered to be important. (b) Volume rendering of a breast dataset. The tumor, highlighted in cyan, corresponds to a low persistent feature in the input. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)