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Technical Section

Quasi-developable surface modeling of contours with curved triangular patches ☆

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ABSTRACT

Skinning, also called lofting, is a powerful and popular method for modeling complex shapes. A surface modeled by the current skinning techniques nevertheless may be far from being developable, which is an important property desired in the manufacturing industry such as ship-hull, wing and body of aircraft, garment, etc. In this paper, a novel approach to skinning surface modeling is proposed. The proposed method interpolates the given curves with a collection of G^1 continuous self-defined triangular patches, and these patches are assembled together by globally minimizing the integral Gaussian curvature, i.e., the degree of developability. The proposed algorithm has been tested on a set of examples and the test results have demonstrated its promising use in a variety of applications.

1. Introduction

Skinning, also referred to as lofting, is a popular modeling tool for complex surfaces. When designing the geometric shape of a surface, one often starts with a few ordered characteristic curves and a skinned surface is defined to interpolate these curves in the given order [20,29,34,35]. Besides traditional applications in CAD, skinning techniques also have important applications in reverse engineering such as medical imaging and terrain modeling. In practice, in addition to the contour interpolation requirement, it is usually desired that the surface possess some preferred properties, such as visual appearance, minimum surface area, maximum developability, etc. Among these properties, developability is one particularly desired in manufacturing industries of ship hull [20,32], automobile, garment [44,46], leather [19], paper-craft [24,28] and others, where the shape of a product should be developable or nearly developable so that it can be made from 2D patterns [18] by stamping, welding, sewing, or other means. Although there are known skinning algorithms that use NURBS technique to model and represent a developable surface to interpolate contours, they all put stringent geometric constraints on the given contours, which are usually highly nonlinear and severely restrictive. In industrial applications, it is often required that the given cocntours are fixed (as they carry the design intent), while skinning surface is sought after that can maximize the degree of developability. Unfortunately, current NURBS or T-spline based skinning techniques [20,29,34,35] do not support this requirement. As a feasible alternative (cf. [46]), a series of discrete developable patches such as triangular or quadrilateral patches can be assembled together to collectively represent the final interpolatory surface. By carefully defining the scheme of assembling these elementary patches, the developability of the final surface can be tremendously improved. In order for this assembly method to work, two problems must be solved: (a) how to define suitable elementary triangular (or quadrilateral) patches for tiling that not only interpolate the given contours but also guarantee smooth connection (e.g., G^1 continuity among neighboring patches, as in this paper); and (b) after the scheme of elementary patch is defined, how to determine an optimal assembly of tiling patches i.e., the connection correspondence. As illustrated in Fig. 1, different connection correspondences could result in varied shapes with drastically different degrees of developability, even with a same construction scheme of elementary patch.

Solving the above two problems is not trivial. In this paper, we present a simple-to-implement construction algorithm for achieving an optimal G^1 skinning surface in terms of integral Gaussian curvature, which is used as the measure of the degree of developability. One major motivation behind the proposed method is the observation that, since the design intent is mostly encapsulated by the characteristic curves, a good skinned surface should be as simple and flat as possible, as long as the interpolation

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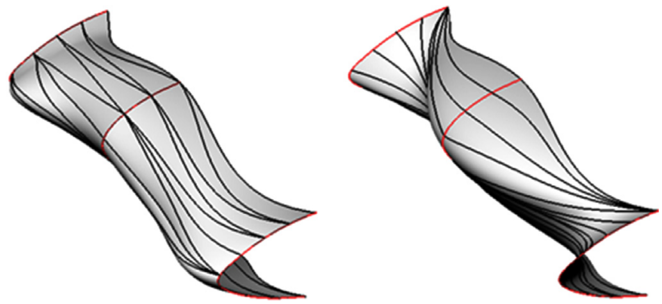

Fig. 1. Two different configurations of assembly of triangular patches, both of which collectively interpolate three contours (red curves). The Gaussian curvature integral of the surface in the left is much smaller than that of the right one; in other words, the surface in the left is more developable. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

requirements are met. In the extreme case, when only G^0 continuity is required, a composite of ruled surfaces between neighboring curves suffices to meet the need. In discrete form where the input curves are given as polygonal chains, this composite surface is made of triangles whose vertices all fall on the curves. To cater to the G^1 continuity and the given normal vector constraints, flat triangles are replaced by us with curved triangles, which are to be referred to as *Triangular Patch* (TP). Basically, a TP is a surface that is completely determined by its three corners and the tangents assigned at them. We call a skinned surface made of TPs a *Patch Triangulation*.

In order to construct a skinned surface with an optimal patch triangulation in terms of developability, we formulate it as the following variational optimization problem: Given N ordered smooth parametric curves $C_1 = f_1(t), C_2 = f_2(t), \ldots, C_N = f_N(t)$, and a fairness function $E(S)$ (which measures the surface developability), find a composite G^1 surface S made of TPs that (i) exactly interpolates the N curves C_i, and (ii) minimizes the integral $E(S)$. In this paper, a construction algorithm is proposed for the above variational problem, involving the following three steps:

(1) Adaptive sampling is performed on each input curve C_i, splitting it into piecewise curve-segments denoted as H_i^j, $1 \le i \le N, 1 \le j \le i$, where m_i is the number of sampling points on curve C_i; each H_i^j is defined by the parametric equation of curve C_i in the interval $[t_j, t_{j+1}]$, which are parameters of the corresponding sampling points on C_i.
(2) For every three sampling points (lying on two adjacent curves C_i and C_{i+1}), devise a constructive scheme that defines a curved triangular patch interpolating them. The crux problem of how to guarantee the required G^1 continuity among the neighboring patches is solved in this step.
(3) Based on the constructive scheme in step (2), find the optimal assembly of TPs by solving the optimal correspondence problem in terms of the fairness function $E(S)$.

The last two steps together are the major contribution of this paper. Note that TPs in an assembly in (3) are not arbitrary—each of them is sandwiched between two adjacent curves C_i and C_{i+1}. Apparently, the solution to the correspondence problem in step (3) is influenced by the construction scheme in step (2) and vice versa. Inspired by the idea of the work [46], the problem in step (3) is transformed into a shortest path-finding problem. The differences between our work and [46] lie in two aspects: (a) we extend the original algorithm [46] to support multiple curves rather than only two in [46] (note that, as the connection correspondences between each pair of strips influence each other, we cannot independently apply the method of [46] for every pair of adjacent strips and then simply stack them together); and (b) instead of planar triangles, curved triangular patches are adopted as elementary tiling elements, which ensure the desired G^2 continuity.

2. Related work

From differential geometry, a surface is developable if its Gaussian curvature is zero everywhere on the surface [9]. While there is a large body of research works in the general area of developable surfaces, few are on how to interpolate multiple contours with a developable surface, the subject of this paper. In the following, we review some key related works in this subject.

Many works [1–3,12,15,21,25] have focused on how to design one developable surface or a series of developable composite Bezier patches to interpolate two arbitrary curves. In this group, most of attention has been paid to how to manipulate or perturb the positions of the control points [47] of the input curves to satisfy the nonlinear constraints due to developability. Auman [1] established the condition under which a developable Bezier surface can be constructed with two boundary curves. The boundary curves in his method are restricted to lie in parallel planes, and their degree is restricted to three. In the work of Frey and Bindschadler [15], the results of Auman are extended by generalizing the degree of the directions. Maekawa and Chalfant [25] extended Auman's algorithm to B-Spline curves by segmenting the original B-Spline curves into multi-segment Bezier curves; so, in essence, they still require the two directrices to lie in parallel planes. Chu and Sequin [12] proposed a method to construct a developable Bezier patch between two boundary curves. In their method, after one boundary curve is arbitrarily specified, five more degrees of freedom are available for the second boundary curve of the same degree. Auman [2,3] utilized the *De Casteljau* algorithm to design developable Bezier surfaces through a Bezier curve of arbitrary degree and shape. All of these methods require solving highly nonlinear algebraic equations, and, most crucially, the resultant surface is difficult to predict, usually with unsatisfactory shapes. Although the resultant surface may be fully developable, the two input curves are however usually altered, which is not allowed in most applications.

Researchers also studied the problem of using projective geometry (also called dual approach) for developable surface interpolation and approximation [6,7,37,39]. As a developable surface can be considered the envelope of a single-parameter family of planes, control planes determined by one parameter can be defined to represent a developable surface. In this manner, a plane is regarded as one point in projective geometry, and a developable surface can be designed using control planes akin to designing a polynomial curve using control points in the Euclidean space. Thus, the designed surface has all the characteristics of the existing methods for curve design. In essence, the control planes can be viewed as the input to interpolation, which though cannot be directly applied in skinning applications. Leopoldseder and Pottman [22] used a series of smoothly linked circular cones to model a given surface, in which each pair of rulings and tangent planes are figured out to construct cones. Their later works can be used to interpolate a point cloud for applications in reverse engineering [10,38]. However, they cannot be directly applied to our developable skinning surface problem.

Recently, algorithms based on discrete data representation have been proposed for modeling developable mesh surfaces, owing to the rapid increase of computing power and popularity of 3D meshes. In this manner, triangle or quad meshes are sought to achieve maximum developability, with the given interpolating constraints satisfied [11,16,24,40,42–46,48]. Usually, the positions