Available online at www.sciencedirect.com
SCIENCE DIRECT
Computers & Graphics 29 (2005) 594–605
COMPUTERS & GRAPHICS
www.elsevier.com/locate/cag

Technical section

Rational cubic spline interpolation with shape control

Zulfiqar Habib[a], Muhammad Sarfraz[b,*], Manabu Sakai[a]

[a]Department of Mathematics and Computer Science, Kagoshima University, Koorimoto 1-21-35, Kagoshima 890-0065, Japan
[b]Department of Information and Computer Science, King Fahd University of Petroleum and Minerals, KFUPM # 1510, Dhahran 31261, Saudi Arabia

Abstract

A rational cubic spline, with shape control parameters, has been discussed here with the view to its application in computer graphics. It incorporates both conic sections and parametric cubic curves as special cases. An efficient scheme is presented which constructs a curve interpolating a set of given data points and allows subsequent interactive alteration of the shape of the curve by changing the shape control and shape preserving parameters associated with each curve segment. The parameters (weights), in the description of the spline curve can be used to modify the shape of the curve, locally and globally. The rational cubic spline retains parametric C^2 smoothness. The stitching of the conic segments also preserves C^2 continuity at the neighboring given points. An exact derivative as well as a very simple distance-based approximated derivative schemes are presented to calculate control points. The curve scheme is interpolatory and can plot parabolic, hyperbolic, elliptic, and circular splines independently as well as segments of a rational cubic spline. We discuss complex cases of elliptic arcs in space and introduce intermediate point interpolation scheme which can force the curve to pass through a given point between any segments.
© 2005 Elsevier Ltd. All rights reserved.

Keywords: Curves and surfaces; Rational cubic spline; Conic; Shape preserving

1. Introduction

A common problem, in computer graphics, is to design a curved outline by stitching small pieces of curves together. Piecewise rational cubic spline functions provide powerful tools for designing curves, surfaces, and some analytic primitives such as conic sections that are widely used in engineering design and various computer graphics applications [1,2]. These applications may represent a font outline [3,4], a rounded corner of an object, or may be a smooth fit to given data [5,6]. Several curve segments that compose a desired curve outline can have different mathematical descriptions. For example, the outline of the character "S" appears to be composed have straight lines, conics, and cubics. A single mathematical formulation for the precise definition of various types of geometry shapes is one of the major advantages of the rational cubic spline functions. Our research aims to develop a piecewise parametric curve representation scheme capable of representing shape outlines.

In [7], C^1 rational cubic splines with exact derivatives at their control points were used. We introduce a similar interpolant with a very simple distance-based approximated derivative scheme and achieve fine results. Our scheme is also simpler than the area-based derivative scheme in [8]. Our research describes a parametric C^1 and C^2 rational cubic spline representation possessing a family of shape control parameters. This family of shape

*Corresponding author. Tel.: +966 3 860 2763; fax: +966 3 860 1562.
E-mail addresses: habib@eniac.sci.kagoshima-u.ac.jp (Z. Habib), sarfraz@kfupm.edu.sa (M. Sarfraz), msakai@sci.kagoshima-u.ac.jp (M. Sakai).

0097-8493/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.cag.2005.05.010

parameters has been utilized to produce straight line segments, conics, and cubics. The ability to maintain a reasonable amount of continuity (C^2) between conic and cubic arcs, estimated end derivatives, conic (circular, elliptical, parabolic, and hyperbolic) splines, circular arcs for given radius or center, elliptic arcs in space and intermediate point interpolation are further achievements in this research. In [7,9], the end derivatives are determined by the user, which is not convenient. Moreover, conics were not discussed at all. We have estimated most suitable end derivatives for more pleasing results. In [10], cubic and conic segments are joined with G^1 continuity which is not acceptable for some practical applications. The intermediate point interpolation scheme and circular arcs, presented in [11], are not practical as the space curves and exact circular arcs in that way. Ref. [12] offered intermediate point interpolation scheme with C^0 continuity at neighborhood points. Meek et al. [13] presented G^1 continuity in their recent research work on constrained guided curve scheme. They used a rational quadratic function. We use a rational cubic function and achieve better continuity (C^2). In [14], a rational quadratic spline is used to represent a circular spline. We have used a rational cubic spline to achieve the same result.

We have used a very simple algorithm for any type of planer or space curve with parallel or non-parallel end tangents. Our scheme can generate exact circular and elliptical arcs. We have applied degree elevation techniques on rational quadratic spline as mentioned in [15–17]. A Non-uniform rational B-spline (NURBS) representation of an ellipse is given in [17]. We have improved this technique to handle any type of elliptic arcs, even space arcs. In addition, our scheme has the following properties, which may lead to a more useful approach to curve and surface design in CAGD.

- The curve has C^2 continuity between the rational cubic arcs and between cubic and conic arcs.
- Most suitable end derivatives are estimated.
- A distance-based approximated derivative scheme is also used to compute the required control points. Tangent vectors vary continuously along the curve preserving C^1 continuity.
- Any part of the rational cubic spline can represent a conic (with exact circle and ellipse) or a straight line segment using the same interpolant.
- An intermediate point interpolation scheme has been introduced for use in guided curves.
- Our scheme can handle any kind of elliptic arc in space.
- Most of the results are visualized with their associated curvature plots for easy comparison between different schemes.
- The benefit of using such curves in the design of surfaces, in particular surfaces of revolution and swept surfaces, is the control of unwanted flat spots and undulations.

This paper has been organized in such a way that a parametric rational cubic spline scheme is considered in the next section. The analysis of the designing curve is presented in Section 3. The determination of the approximated and exact tangents at the given points is explained in Section 4. In this section, we also present a scheme to calculate the end derivatives (tangents). We discuss conditions for conics and straight line segments in Section 5. This section also covers all types of circular and elliptical arcs in space and introduces a very powerful method for intermediate point interpolation. Examples are discussed in Section 6. Finally, our conclusions are presented in the last section.

2. The rational cubic spline

The cubic spline is the spline of the lowest degree with C^2 continuity. C^2 continuity meets the needs of most problems arising from engineering and mathematical physics. Rational cubic spline functions of lower degree are numerically simple, stable, and fundamental of all rational space curves. Let $\boldsymbol{F}_i \in \mathbb{R}^m, i = 1, \ldots, n,$ be a given set of points at the distinct knots $t_i \in \mathbb{R}$, with unit interval spacing. Consider a first degree parametric piecewise rational function for the straight line segment between $\boldsymbol{F}_i$ and $\boldsymbol{F}_{i+1}$

$$\boldsymbol{L}(t) \equiv \boldsymbol{L}_i(t) = \frac{(1-s)\alpha_i \boldsymbol{F}_i + s\beta_i \boldsymbol{F}_{i+1}}{(1-s)\alpha_i + s\beta_i}, \tag{2.1}$$

where

$$s = \frac{t - t_i}{h_i}, \quad h_i = t_{i+1} - t_i.$$

We apply degree elevation formula [1, p. 104] to get quadratic rational Bézier function

$$\boldsymbol{Q}(t) \equiv \boldsymbol{Q}_i(t) = \frac{(1-s)^2\alpha_i \boldsymbol{F}_i + s(1-s)\gamma_i \boldsymbol{U}_i + s^2\beta_i \boldsymbol{F}_{i+1}}{(1-s)^2\alpha_i + s(1-s)\gamma_i + s^2\beta_i}, \tag{2.2}$$

where $\boldsymbol{U}_i$ may be taken as the point of intersection of tangents at $\boldsymbol{F}_i$ and $\boldsymbol{F}_{i+1}$ (see Fig. 1). Applying again degree elevation, we get rational cubic Bézier function

$$\boldsymbol{P}(t) \equiv \boldsymbol{P}_i(t) = \frac{(1-s)^3\alpha_i \boldsymbol{F}_i + s(1-s)^2(\alpha_i+\gamma_i)\boldsymbol{V}_i + s^2(1-s)(\beta_i+\gamma_i)\boldsymbol{W}_i + s^3\beta_i \boldsymbol{F}_{i+1}}{(1-s)^2\alpha_i + s(1-s)\gamma_i + s^2\beta_i}, \tag{2.3}$$

Download English Version:

https://daneshyari.com/en/article/10336520

Download Persian Version:

https://daneshyari.com/article/10336520

Daneshyari.com