ARTICLE IN PRESS

Graphical Models xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Graphical Models

journal homepage: www.elsevier.com/locate/gmod

Computing a compact spline representation of the medial axis transform of a 2D shape

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ARTICLE INFO

Article history:
Received 1 March 2014
Accepted 17 March 2014
Available online xxxx

Keywords:
Shape modeling
Medial axis transform
Spline
Curve fitting

ABSTRACT

We present a full pipeline for computing the medial axis transform of an arbitrary 2D shape. The instability of the medial axis transform is overcome by a pruning algorithm guided by a user-defined Hausdorff distance threshold. The stable medial axis transform is then approximated by spline curves in 3D to produce a smooth and compact representation. These spline curves are computed by minimizing the approximation error between the input shape and the shape represented by the medial axis transform. Our results on various 2D shapes suggest that our method is practical and effective, and yields faithful and compact representations of medial axis transforms of 2D shapes.

1. Introduction

The notion of the medial axis transform was first introduced by Blum [1] as an intrinsic shape representation. The medial axis of an object $\mathcal{O}$ is the set of interior points having at least two closest points on the boundary $\partial\mathcal{O}$ of $\mathcal{O}$. In the 2D space, each point on the medial axis is the center of a circle, namely a *medial circle*, which is the maximal inscribed circle contained in $\mathcal{O}$ and tangent to $\partial\mathcal{O}$ in at least two points. To encode the complete shape information of the object, each point on the medial axis is assigned with the radius, which could be 0, of its associated medial circle. Therefore, a radius function could be defined on the medial axis. The medial axis coupled with a radius function is referred to as the *medial axis transform* (MAT). Each point in the MAT, called *medial point*, has three dimensions, which indicates its 2D position and the radius. The MAT is a complete shape representation in the sense that the object boundary can be reconstructed exactly from its MAT as the envelope of all the medial circles.

The MAT encodes rich information of a shape, such as local thickness, symmetry and its part structure, which is not possessed by alternative boundary surface representations. Therefore the MAT has been used extensively in a wide spectrum of applications, including shape analysis [2], shape deformation [3] and artistic rendering [4]. Detailed discussions on properties and applications of the MAT can be found in the book [5].

The MAT, on the other hand, is well-known suffering from the *instability* problem: small variations of the shape boundary may yield a large change to its MAT. While boundary noise is ubiquitous in data acquisition due to errors introduced in scanning, sampling and other numerical processing, the medial axis thus often has excessive geometric complexity and pathological topology, rendering it generally useless in practice unless it is cleaned up. A lot of existing algorithms have been developed to resolve the instability issue. As a common practice, unstable branches of the MAT induced by boundary noise are pruned based on certain measures [6–8]. Different criteria have been introduced to characterize the difference between the original shape and the reconstructed shape, or describe the remaining stable MAT with some intrinsic

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http://dx.doi.org/10.1016/j.gmod.2014.03.007
1524-0703/

Please cite this article in press as: Y. Zhu et al., Computing a compact spline representation of the medial axis transform of a 2D shape, Graph. Models (2014), http://dx.doi.org/10.1016/j.gmod.2014.03.007

measures [9]. These methods focus on simplifying the topology of the MAT by pruning unstable branches, producing a topologically clean MAT which is nevertheless still represented by a large number of sample medial points. This is partially due to the prevailing choice taking the union of the sample medial circles (or medial spheres in 3D) as an approximation when evaluating the approximation error during both pruning stage and shape reconstruction stage. As a consequence, a large number of medial circles are often necessary to attain a good approximation to the MAT [10,11].

To achieve a smooth representation and further reduce the geometric complexity of the MAT, we propose to represent the MAT as spline curves. See Fig. 1 for a comparison of the two shape representations. The medial axis also possesses piecewise C^2 continuity in each medial branch. Although a similar smooth representation of the MAT has been used for modeling and segmentation purposes [12], a fully automatic way of obtaining a smooth MAT for an arbitrary shape is still missing.

In this work, we propose a complete pipeline which automatically computes a *stable and compact* medial axis transform which accurately approximates an arbitrary 2D shape. Given an error threshold $\hat{\varepsilon}$, our algorithm guarantees that the Hausdorff distance between the original shape and the reconstructed shape is at most $\hat{\varepsilon}$. Our method involves the pruning of unstable medial branches with an error-driven filtering process and the computation of a compact and accurate spline approximation to the MAT.

Compared to other works on medial axis computation, our algorithm possesses the following advantages:

- *Topological filtering with error control*: Our filtering process is guided by a user-defined error threshold $\hat{\varepsilon}$ to ensure approximation accuracy while removing noisy, unstable branches as much as possible.
- *Computing a compact geometric representation*: We use spline curves to approximate the MAT, resulting in a highly compact representation. An optimization process is developed to make sure that the reconstructed boundary best fits the input shape, meeting a user specified error tolerance.

This paper is organized as follows. We start with a brief review of the previous work related to medial axis computation in Section 2. We then define the piecewise smooth medial representation in Section 3. In Section 4, we introduce our main algorithm; the implementation details are then provided in Section 5. We present experimental results in Section 6 and finally conclude the paper in Section 7.

2. Related work

There is a vast amount of research studies about medial axis computation and representation. Here, we will review only those which are in close relation to our work.

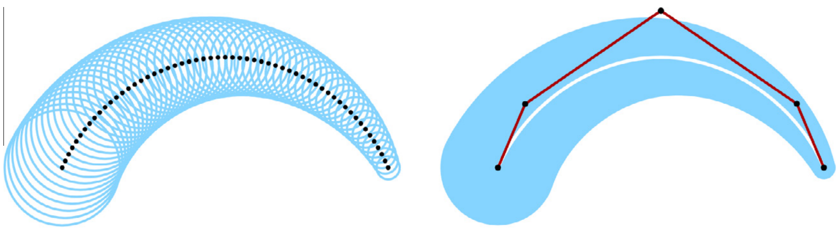
2.1. Medial axis computation

Exact medial axis computation is possible only for simple or special shapes, such as polyhedra [13,14]. For free-form shapes, medial axis approximations are widely used in practice. There are several main approaches to computing the medial axis approximation: pixel or voxel-based methods that compute the medial axis using a thinning operation [15]; methods based on distance transform [7,16–18], often performed on a regular or adaptive grid; the divide-and-conquer methods [19]; the tracing approaches [20] and the Voronoi diagram (VD) based methods [6,21–24].

Among these, the VD based approach stands out due to its theoretical guarantee and efficient computation. As a preprocessing step, we obtain an initial discrete medial axis of a shape using the VD based algorithm. The VD based method assumes that the boundary of an input shape $\mathcal{O}$ is a smooth curve and is sampled by a dense discrete set $\mathbf{P}$ of points (Fig. 2a), with the sampling density determined by the local feature size [21] in order to capture the boundary topology correctly. The Voronoi diagram of $\mathbf{P}$ is computed and the Voronoi vertices interior to $\mathcal{O}$ are taken to approximate the medial axis of Voronoi diagram (Fig. 2b), since a point on the Voronoi diagram is also characterized by having at least two closest points among the sample points.

2.2. Handling instability

Many studies have been conducted to understand and resolve the instability problem of the MAT. We review here



(a) (b)

Fig. 1. Given a shape swept by a moving circle, its medial axis transform is approximated as a sample set of 57 medial circles. (a) The approximate shape is the union of the medial circles. (b) On the other hand, to achieve the same accuracy with spline medial axis approximation, we need only 5 control points. The control polygon of the spline curve is shown in red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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