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Isogeometric segmentation. Part II: On the segmentability of contractible solids with non-convex edges

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ABSTRACT

Motivated by the discretization problem in isogeometric analysis, we consider the challenge of segmenting a contractible boundary-represented solid into a small number of topological hexahedra. A satisfactory segmentation of a solid must eliminate non-convex edges because they prevent regular parameterizations. Our method works by searching a sufficiently connected edge graph of the solid for a cycle of vertices, called a cutting loop, which can be used to decompose the solid into two new solids with fewer non-convex edges. This can require the addition of auxiliary vertices to the edge graph. We provide theoretical justification for our approach by characterizing the cutting loops that can be used to segment the solid, and proving that the algorithm terminates. We select the cutting loop using a cost function. For this cost function we propose terms which help to select geometrically and combinatorially favorable cutting loops. We demonstrate the effects of these terms using a suite of examples.

1. Introduction

Isogeometric analysis (IGA) is an approach to the solution of partial differential equations in which the functions used to approximate solutions are the same as those used to parameterize the geometry [9,6]. IGA aims to bridge the gap between CAD and analysis communities and their technologies, and in particular, to automate as much as possible the process of translating between CAD and analysis objects.

In the *boundary representation* (BRep), a solid object is represented by a structure consisting of vertices, edges and faces. The faces are trimmed surfaces describing the boundary of the solid. In order to prepare a boundary-represented solid for IGA, it is necessary firstly to decompose a solid into blocks that are *topological hexahedra*, i.e., sufficiently smooth (say, continuously differentiable) embeddings of a cube, and secondly to construct a suitable parameterization for each block.

There is much existing work addressing the parameterization question under appropriate conditions. A solid is parameterized by a cube using B-splines in [16,19] and using T-splines in [23,24]. In [22], 3D models are parameterized by polycubes. T-spline surfaces or solids are constructed from given quad- or hex-meshes in [18]. The methods of [13] can be used to parameterize one solid by another using harmonic mappings. Swept volumes are parameterized in [1].

There is a rich theory for the segmentation of polyhedra into convex polyhedra [3,4,2]. Techniques have also been developed for parameterizing multiple patches of complex, arbitrary solids [20], relying on a predefined segmentation. In [15,14] a technique for segmentation and parameterization is developed for triangulated solids, allowing for interior features. The method of [17], also for triangulated

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solids, uses T-splines and allows for solids of arbitrary genus.

The authors of [10] initiated the development of a technique to produce an IGA-suitable decomposition of a boundary-represented solid into topological hexahedra. The goal is to produce a small number of topological hexahedra. The method was constrained to solids with only convex edges, that is, for any edge of the solid, the incident faces meet at a convex interior angle at every point of the edge. The existence of a non-convex edge creates significant restrictions on the decomposition. A satisfactory segmentation must cut the solid at this edge, because a single differentiable trivariate patch can only have convex edges. Isogeometric segmentation of solids containing non-convex edges has not previously been considered.

The present paper continues research on IGA-suitable segmentation of a solid into topological hexahedra, by extending the approach of [10] to apply to solids with non-convex edges. We recursively decompose the edge graph of the solid into smaller subgraphs, by searching for a *cutting loop* in the edge graph. At each step of the decomposition we make sure that the subgraphs have less non-convex edges than the previous graph. Eventually the solid is decomposed into new solids with only convex edges. Then, as in [10], these solids can be further decomposed into hexahedra. In Fig. 1 we show the first few steps of our segmentation of a model with non-convex edges.

Non-convex edges make the search for a cutting loop more complex. They impose additional geometric constraints on valid cutting loops. Sometimes it is necessary to add new vertices to the edge graph of the solid. We provide a proof that a cutting loop can be found through any given non-convex edge. We provide a method of choosing among multiple cutting loops, which is a combination of geometric and topological criteria, and depends on choices for several parameters. Our cost function is more complex than that of [10], and we show how our new additions are important for finding reasonable decompositions.

In Section 2 we provide our assumptions about the solid and give some definitions and a way to test for a non-convex edge. We recall the *Isogeometric Segmentation Problem* as stated in [10]. Section 3 covers our approach to solids containing non-convex edges. We give a geometric criterion for the validity of a cutting loop. We provide our algorithm for splitting the edge graph of a solid that contains at least one non-convex edge.

In Section 4 we prove that there always exists a valid cutting loop through a given non-convex edge. As a consequence, combining our algorithm with that from [10] we are capable of reducing a solid to topological hexahedra. Section 5 describes our cost function for selecting among multiple cutting loops. In Section 6 we study several examples, showing how the geometric part of our cost measures the deviation from planarity, and applying several sets of parameters to several examples to explore how the choice of parameters affects the final number of topological hexahedra in the decomposition. In Section 7 we summarize our findings and discuss the outlook.

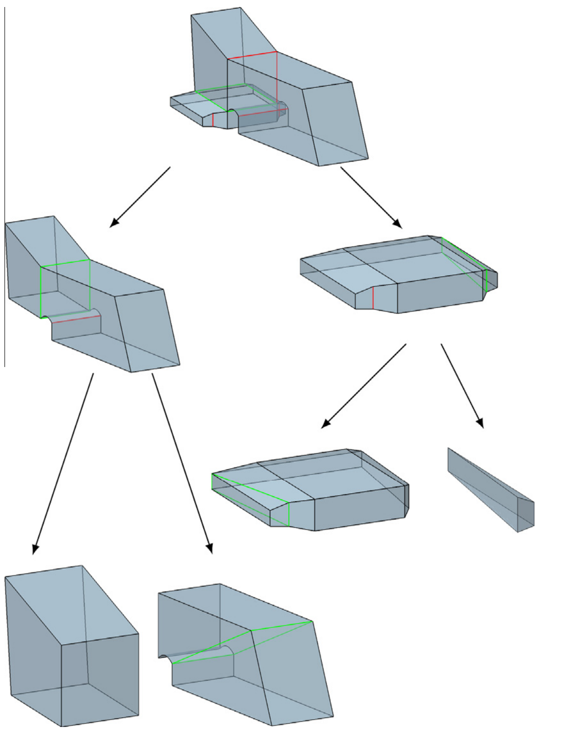

Fig. 1. Early steps in the segmentation of a solid with non-convex edges. See Fig. 11 for a complete segmentation into hexahedra and triangular prisms.

2. Preliminaries

In this section we describe our assumptions about the input solid and state the *Isogeometric Segmentation Problem*.

2.1. Assumptions

We consider a solid object $\mathcal{S}$ given by its boundary representation (BRep). We briefly recall from [10] that the solid object is defined as a collection of vertices, edges and faces. We assume that the edges are represented by NURBS curves, and that the faces are represented as trimmed NURBS patches. Thorough descriptions of NURBS, trimmed NURBS surfaces, and BRep are presented in, e.g., [5].

The edge graph $\mathcal{G}(\mathcal{S})$ of the solid is obtained by restricting the consideration to only the vertices and the edges of the solid.

Consider an edge $\mathbf{e}$, and its two neighboring faces $\mathbf{f}_1$ and $\mathbf{f}_2$. The normal plane at a point $\mathbf{p}$ of $\mathbf{e}$ intersects $\mathbf{f}_1$ and $\mathbf{f}_2$ in two planar curve segments. It also contains the two outward normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ of $\mathbf{f}_1$ and $\mathbf{f}_2$ at $\mathbf{p}$ respectively. Let $\mathbf{t}_1$ and $\mathbf{t}_2$ be the two tangent vectors of the two planar curve segments in $\mathbf{f}_1$ and $\mathbf{f}_2$ respectively (oriented such that they point away from the edge), see Fig. 2. Recall again from [10] that the edge is said to be *convex at* $\mathbf{p}$ if $\mathbf{n}_1$ and $\mathbf{t}_1$ do not (trivially or strictly) separate $\mathbf{n}_2$ and $\mathbf{t}_2$, i.e., the convex cone generated by $\mathbf{t}_1$ and $\mathbf{n}_1$ only intersects