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As-rigid-as-possible spherical parametrization

Chunxue Wang, Zheng Liu *, Ligang Liu

School of Mathematical Sciences, University of Science and Technology of China, Hefei 230026, China

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ABSTRACT

In this paper, we present an efficient approach for parameterizing a genus-zero triangular mesh onto the sphere with an optimal radius in an *as-rigid-as-possible* (ARAP) manner, which is an extension of planar ARAP parametrization approach to spherical domain. We analyze the smooth and discrete ARAP energy and formulate our spherical parametrization energy from the discrete ARAP energy. The solution is non-trivial as the energy involves a large system of non-linear equations with additional spherical constraints. To this end, we propose a two-step iterative algorithm. In the first step, we adopt a local/global iterative scheme to calculate the parametrization coordinates. In the second step, we optimize a best approximate sphere on which parametrization triangles can be embedded in a rigidity-preserving manner. Our algorithm is simple, robust, and efficient. Experimental results show that our approach provides almost isometric spherical parametrizations with lowest rigidity distortion over state-of-the-art approaches.

1. Introduction

Parametrization is essential for geometry processing and has wide applications in various fields, including texture mapping, texture synthesis, detail transfer, mesh completion, remeshing, surface approximation, scattered data fitting, and morphing, etc.

The problem of mesh parametrization is to compute a one-to-one mapping from a given mesh to a suitable domain. The most commonly used mesh is triangular mesh, and the mappings are required to be at least piecewise linear, so we only need to compute the vertex coordinates.

The parametrization of a closed genus-zero manifold triangular mesh preferably is done on its natural domain: the spherical domain. Many applications are quite sensitive to discontinuities in the parametrization, or cannot tolerate them at all. The big advantage of the spherical domain over the planar one is that it allows for seamless, continuous parametrization of genus-zero models, and there are a large number of such models in use [1].

As we have known, the common goal of parametrization is to find a mapping which minimizes some metric distortion of the original mesh. There are various types of parametrization methodologies, such as authalic (area-preserving) mapping, conformal (angle-preserving) mapping, isometric (length-preserving) mapping, and some combination of these [2]. In this paper, we aim to parameterize a genus-zero triangular mesh onto a sphere in an as-rigid-as-possible (ARAP) manner.

Specifically, we extend the ARAP planar parametrization method [2,3] to spherical domain. In the planar case, the ARAP parametrization [2] minimizes an "intrinsic" deformation energy function which can be expressed in terms of the singular values of the Jacobian of the parametrization. Unfortunately, the extension to the sphere is not straightforward. Furthermore, in *sharp* contrast to the planar case, we need to optimize a radius of the sphere on which parametrization triangles can be embedded on in a rigidity-preserving manner.

* Corresponding author.
E-mail address: liu.zheng.jojo@gmail.com (Z. Liu).

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After we formulate the smooth ARAP spherical parametrization energy, we propose the discrete variational description of the energy. We use a two-step iterative algorithm to solve the non-linear problem with additional spherical constraints. An optimal radius of the sphere is obtained to guarantee the rigidity of the parametrization. Experimental results show that our ARAP parametrization approach always gives the lowest rigidity distortion compared with state-of-the-art approaches [4–7].

2. Related work

2.1. Spherical parametrization

The problem of mesh spherical parametrization is mapping a piecewise linear surface with a discrete representation onto a spherical surface. There are many methods of spherical parametrization proposed in the past years. We refer the interested reader to [8,1] for a survey of the state-of-the-art in spherical parametrization research. Many of these methods are very similar to those of mapping simple meshes onto planar domain, whereas some of the linear methods become non-linear versions.

The simplest way to map a closed triangular mesh to the sphere is to reduce the problem to the planar case. Haker et al. [9] used a method which mapped the given genus-zero mesh into the plane and then used stereographic projection to map it to the sphere. Thus, it is more natural to parameterize the mesh directly on the sphere without going back and forth to the plane. Several methods for direct parametrization on the sphere exist. Gotsman et al. [5] showed a nice relationship between spectral graph theory and spherical parametrization, and embedded simple meshes onto the sphere by solving a quadratic system. Unfortunately, there was no analysis of the degrees of freedom in the various spherical embedding, the system was also quite slow even for a few hundred vertices. Saba et al. [10] provided an efficient optimization method combined with an algebraic multigrid technique to get the solution of the large system of non-linear equations in [5]. Li et al. [6] minimized the discrete harmonic energy to map a genus-zero surface to the unit sphere with good shapes which was suitable for surface fitting with PHT-splines.

Some spherical parametrization approaches have focused on directly optimizing the metric distortion. These approaches require extensive computation, due to the distortion measures are usually highly non-linear. Gu and Yau [11] gave an important point that a harmonic spherical map is conformal, then they proposed an iterative method which approximated a harmonic map without splitting. Other spherical conformal mappings are proposed in [4,12,13]. Sheffer et al. [14] proposed a method which measured angle distortion directly. They formulated a set of necessary and sufficient conditions for the spherical angles of the triangulation to form a valid spherical triangulation. The resultant mappings of all methods above are almost angle-preserving. However, the area distortion of these methods should be very large, and the area distortion is necessary to consider. Praun and Hoppe [15] used a coarse-to-fine solving scheme to iteratively optimize the $\mathcal{L}_2$ stretching energy [16] defined piecewise on the triangluar mesh. Zayer et al. [17] proposed a curvilinear spherical parametrization which better reduced area-distortion efficiently. Wan et al. [7] utilized the distortion energy [18] and presented an efficient hierarchical optimization scheme minimizing angle and area distortions. However, we find that little work has been done on rigidity-preserving in spherical parametrization, thus we propose a method using the ARAP energy to parameterize the surface in a rigidity-preserving manner.

Recently, some basic flows have been used to evolve surface geometry. Both are classic mean-curvature flow derivations (see [19] for a modern treatment of the continuous formulation, and [20] for the finite-elements discretization) [21] modified the flow and presented empirical evidence that does it define a stable surface evolution for genus-zero surfaces, but that the evolution converges to a conformal parametrization of the surface onto the sphere. However, discrete surface Ricci flow theory was developed by Chow and Luo [22] and a computational algorithm was introduced in [23]. With spherical geometry, Ricci energy is not strictly convex but converges to a local optimum [22,24,25] presented a framework for spherical parametrization using Euclidean Ricci flow, facilitating efficient and effective surface mapping.

2.2. ARAP energy

The ARAP energy is very important in geometry processing and has wide applications in editing [3], morphing [26,27], simulation [28], and planar parametrization [2,29]. The energy measures the sum of distance between deformation differentials and their corresponding rotation group [28]. Chao et al. [28] analyzed the relationship between the ARAP energy and standard elastic energy which commonly used in simulation. Local rigidity can be seen as the governing principle of various surface deformation models, and Sorkine et al. [3] used the ARAP energy to preserve the local-details of vertex's one-ring neighbors for surface modeling. The principle of ARAP deformation was successfully applied to shape interpolation [26,27] to determine intermediate shape path such that the deformation from source to target appears as rigid as possible. Liu et al. [2] used the ARAP energy for planar parametrization of disk-topology surfaces, they parameterize the mesh to the plane in a rigidity-preserving manner for each individual triangle. Myles and Zorin [29] described a method for finding cone locations, and extended the ARAP parametrization of disk-topology surfaces to general surfaces, which yielded seamless parametrization with low metric distortion.

3. Contribution

We extend the ARAP planar parametrization of disk-topology surfaces [2] to spherical parametrization of closed genus-zero mesh. This poses the problem to find an optimal local transformation for each individual mesh element, then stitch the transformed elements into a coherent

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