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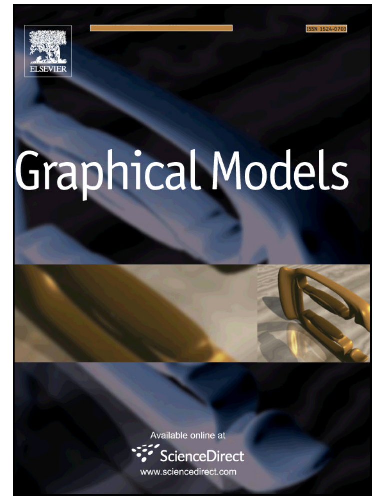

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Precise Continuous Contact Motion for Planar Freeform Geometric Curves

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Abstract

This paper presents an efficient algorithm for generating a continuous precise contact motion between planar geometric models bounded by piecewise polynomial C^1-continuous parametric B-spline curves. A system of algebraic constraint equations is formulated and then efficiently solved for the two/three-contact configuration between two planar B-spline curves. The result is essentially the same as the generation of configuration space obstacle for a moving curve (with translation and rotation) against a stationary curve. The two-contact motion can be characterized as the intersection curve in the boundary of the configuration space obstacle.

The topology of the reconstructed solution is guaranteed to be correct up to a prescribed tolerance and we demonstrate the effectiveness of the proposed approach using several test examples of continuous contact motion among planar freeform smooth geometric models.

Keywords: Configuration space obstacle, contact motion, freeform geometric models, B-spline curves.

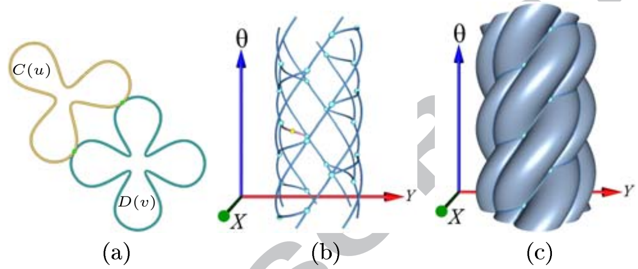

Figure 1: Continuous contact motion between two planar non-convex curves: (a) the contact motion in the work space, (b) the corresponding motion as computed in this work on the boundary of the configuration space obstacle in the $xy\theta$-space (see the pink edge in (b)) and (c) the whole boundary of the C-space obstacle. Note that the angle θ is periodic and the top of the network is continuously connected to the bottom.

1 Introduction

The geometric concept of configuration space is one of the most important tools for reasoning about planning in robotics and automation [8, 11, 19, 20, 22, 23, 24, 28]. Nevertheless, the construction of explicit configuration spaces has been limited to those for low-dimensional cases such as translational motion in 2D or 3D and planar motion with three degrees of freedom (i.e., translation and rotation in the plane). Furthermore, in the motion planning of non-polygonal curved objects, the configuration space is usually bounded by high-degree curves and surfaces, an efficient and precise construction of which has been a non-trivial problem.

Industrial objects are mostly designed with Non-Uniform Rational B-Spline (NURBS) curves and surfaces, which are the de facto industry standard for 3D modeling. Consequently, the contact motion analysis for numerically controlled (NC) milling machines with respect to NURBS models has been one of the main research topics in computer aided design and manufacturing [14]. Because of the requirement of high-precision in manufacturing, solutions based on the polygonal approximation of NURBS models are often not acceptable. It is also a common practice that the NC tool-paths are approximated with NURBS curves in the planning stage.

In this paper, we consider a general contact motion analysis for a planar freeform smooth curve $C(u)$ with respect to a similar but static curve $D(v)$ in the plane. As the moving curve has two degrees of freedom in translation (x, y) and one degree of freedom in rotation θ, the configuration space is a three-dimensional space (i.e., the $xy\theta$-space). The moving curve can be represented as $C(u, x, y, \theta) = R_\theta(C(u)) + (x, y)$, where R_θ is the planar rotation by angle θ.

The configuration space (C-space) obstacle can be bounded by implicit surfaces of the form $F(x, y, \theta) = 0$, which correspond to the contact configurations between $C(u, x, y, \theta)$ and $D(v)$ but with no interpenetration to each other. The boundary of the C-space obstacle consists of many patches of these implicit surfaces, the exact representation of which is quite complicated. Moreover, these surface patches are bounded by intersection curves with other similar such patches. The intersection curves correspond to *two-contact* configurations between $C(u, x, y, \theta)$ and $D(v)$, i.e., the two curves contact tangentially at two different locations: $C(u_i, x, y, \theta) = D(v_i)$, for $i = 1, 2$, where $u_1 \neq u_2$ and $v_1 \neq v_2$. The intersection curve segment has two endpoints, each of which typically corresponds to a *three-contact* configuration between $C(u, x, y, \theta)$ and $D(v)$. The C-space obstacle boundary thus has a layout which can be represented in a graph structure, where each vertex corresponds to a three-contact configuration and certain pairs of these vertices are connected with edges (representing two-contact continuous motions).

Figure 1 shows an example of a continuous contact motion and the corresponding point moving along an edge on the C-space obstacle boundary. Compared with the non-trivial shape complexity of the whole C-space (Figure 1(c)) of the two given planar curves, the two/three contact arrangement of C-space (Figure 1(b)) has a relatively simple network structure of two-contact motion curve segments. The edges and vertices in this graph are two/three-contact points, which can be computed by solving a system of equations of the form: $F_i(x, y, \theta) = 0$, for $1 \le i \le K$, where $K = 2$ or 3. Though the C-space obstacle boundary is composed of high-degree curves and surfaces, we have developed a highly stable algorithm for computing the two-contact motion curves based on a geometric condition that can guarantee the topological structure of the C-space curve arrangement.

One may consider an alternative approach using polyg-