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Accuracy and semantics in shape-interrogation applications

C.M. Hoffmann [a,1], N.F. Stewart [b,*,2]

[a] *Department of Computer Science, Purdue University, West Lafayette, IN 47907-1398, USA*
[b] *Département IRO, Université de Montréal, CP6128, Succ. CentreVille, Montréal, Que., Canada H3C 3J7*

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Abstract

This paper describes two important questions of semantics, in the context of accuracy in shape interrogation. The first of these questions is how to give meaning to an internally inconsistent solid description based on the widely used trimmed-surface boundary representation. The second question is the meaning of a request, made to a numerical method, to find a solution to a problem whose parameter values are uncertain. Answers to these questions are given, inspired in the case of the second question by a now-standard approach in numerical analysis.

Keywords: Robustness; Accuracy; Semantics; Boundary representations; Backward error analysis; Trimmed surfaces

1. Introduction

Shape interrogation is the process of extracting information from a geometric model [1]. In the context considered here, *shape interrogation* may be taken as a

* Corresponding author. Fax: +1 514 343 5834.
E-mail addresses: cmh@cs.purdue.edu (C.M. Hoffmann), stewart@iro.umontreal.ca (N.F. Stewart).
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synonym for *geometry processing*, a term used earlier by Barnhill [1, *Preface*]. Typical examples of shape interrogation are "given two objects, where is their intersection?" or "given a surface, what does its offset look like?" We are concerned here with the semantics of representations of objects and with the accuracy of solutions to shape-interrogation problems.

The problem of semantics of representations is disguised, in the questions quoted above, by the use of the word "given." The difficulty [2,3] is that often in practice the meaning of a given computer representation is far from clear: representations are usually internally inconsistent, due to the use of finite-precision arithmetic, and the use of low-degree curves to represent high-degree surface intersections. Similarly, there are fundamental semantic questions about inaccuracy, which may either be due to the ill condition of problems, or to the instability of methods. These sources of inaccuracy are very different, and consequently, so is the correct approach, in each case, to dealing with it.

Another aspect of accuracy, in the context of shape interrogation, is the definition of how it should be measured. It is a surprising fact that the definition of a metric, to be used to measure error, is missing from a large segment of the literature on robustness of methods in solid modeling.

Appropriate use of the concepts of problem condition, method stability, error metrics, and the relation between error and uncertain data, have been under development for several decades [4,5], and the overall approach has become standard in modern numerical analysis (see, for example [6]). In this paper we will attempt to embed the accuracy and semantics questions of shape interrogation into the standard numerical-analysis approach, and to describe the conclusions to which this leads.

The remainder of the paper is organized as follows. In Section 2, a brief historical discussion is given, to introduce the two major questions of semantics that will be discussed. Section 3 deals with the semantics of inconsistent representations. Following this, Section 4 discusses two questions related to the exactness of geometric data, Section 5 discusses metrics for the measurement of error, and Section 6 deals with the semantics of ill-defined problems. Section 7 is a short section describing the immediate practical consequences of the paper and Section 8 is a short conclusion.

2. Historical background

What do we mean when we use an internally inconsistent representation of a solid for the purpose of shape interrogation? And what do we mean when we ask a method to solve a problem whose parameters' values have not been exactly specified? These two questions of semantics will be the main subject addressed in this paper. First, however, some background will be given. This brief historical discussion will be didactic, rather than comprehensive: its goal is to motivate our answer to the two questions above.

The first significant study of the problem of attaching meaning to computer representations, in the context of shape interrogation, was contained in a series of technical reports by Requicha and Voelcker in the 1970s. In particular, in the seminal