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Adaptive density estimation using an orthogonal series for global illumination

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Abstract

In Monte-Carlo photon-tracing methods energy-carrying particles are traced in an environment to generate *hit points* on object surfaces for simulating global illumination. The surface illumination can be reconstructed from particle hit points by solving a density estimation problem using an orthogonal series. The appropriate number of terms of an orthogonal series used for approximating surface illumination depends on the numbers of hit points (i.e. the number of samples) as well as illumination discontinuity (i.e. shadow boundaries) on a surface. Existing photon-tracing methods based on orthogonal series density estimation use a pre-specified or fixed number m of terms of an orthogonal series; this results in undesirable visual artifacts, i.e. either near-constant shading across a surface which conceals the true illumination variation when m is very small or excessive illumination oscillation when m is very large. On the other hand, interactive user specification of the number of terms for different surface patches is inefficient and inaccurate, and thus is not a practical solution. In this paper an algorithm is presented for automatically determining on the fly the optimal number of terms to be used in an orthogonal series in order to reconstruct surface illumination from surface hit points. When the optimal number of terms required is too high due to illumination discontinuity of a surface, a heuristic scheme is used to subdivide the surface along the discontinuity boundary into some smaller patches, called *sub-patches*, so as to allow a smaller number of terms in the orthogonal series to optimally represent illumination on these sub-patches. Experimental results are presented to show that the new method improves upon other existing orthogonal series-based density estimation methods used for global illumination in both running time and memory requirements.

Keywords: Photon-tracing; Monte Carlo simulation; Probability and statistics; Density estimation

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1. Introduction

Methods for global illumination computation are categorized into two classes: view-dependent methods and view-independent methods [1]. In a view-dependent method, such as the classical ray tracing, a viewpoint is specified by the user and lighting information depending on the viewpoint is computed to synthesize an image. In a view-independent method, such as the classical radiosity, only the lighting information independent of the viewpoint is computed and stored for subsequent image rendering. The lighting information independent of the viewpoint includes all the light energy interactions between surfaces. Multi-pass methods combining both view-independent and view-dependent light information are the state of the art in global illumination computation.

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Although view-independent methods do not provide a complete global illumination solution, they increase the realism of three-dimensional (3D) virtual environments significantly.

Monte-Carlo methods based on particle tracing is an approach towards computing global illumination that is currently under active research [2–17]. There are two stages in a photon-tracing global illumination method: a light transport stage and an illumination reconstruction stage. In the light transport stage, energy-carrying particles are emitted from light sources, traced through the environment, and reflected from surfaces until they are absorbed probabilistically. In the illumination reconstruction stage, illumination on a surface is estimated from the distribution of all particle-hit points on the surface. The process is illustrated in Fig. 1.

Walter et al. [5] formulated the two-stage photon-tracing global illumination method as a *density estimation framework*, based on the fact that illumination reconstruction from particle hits is an instance of the well-known *density estimation* problem in statistics. Several techniques can be found in the statistics literature for solving the density estimation problem [18], some of which have already been applied to solve problems in computer graphics; these include the histogram method [4,19], the kernel method [5,20], and the orthogonal series estimator [3].

The success of a density estimation framework depends on the performance of the density estimation method used in the framework. In this paper we study the application of the orthogonal series estimator in a photon-tracing global illumination method. A critical parameter in an orthogonal series estimator is the number m of terms in a series used to best approximate illumination on a surface patch; this number m should be different for different surfaces, since it depends on the number of samples (i.e. hit points) on the surface as well as illumination discontinuity along shadow boundaries. However, existing density estimation frameworks based on an orthogonal series estimator use a fixed m for all surface patches in a scene, thus causing either an overly flat shading that conceals the true illumination variation when m is relatively small or excessive illumination oscillation when m is relatively large. Moreover, as will be seen later in this paper, the typical values of m vary in a large range. Hence, it is inefficient, and inaccurate, for the user to specify appropriate values of m for different surfaces.

We present a new adaptive orthogonal series estimator that is integrated with a surface subdivision scheme. Invoking established results from statistics, we determine automatically the optimal number of terms for illumination approximation on a surface patch. When this optimal number of terms becomes too large due to significant illumination discontinuity, we subdivide the surface patch into several smaller surface patches, called *sub-patches*, so that the number of terms that should be used on each sub-patch can be kept small, since high-degree polynomials not only take extra time and memory to process but also cause undesirable illumination oscillations. Our experimental results show that this new density estimator outperforms other existing photon-tracing global illumination methods based on an orthogonal series density estimator.

The remainder of this paper is organized as follows: in Section 2 we review some typical methods for solving the density estimation problem in global illumination computation. In Section 3 we consider determining the optimal number of terms used in an estimation function for illumination representation. In Section 4 we present the adaptive nonuniform subdivision scheme. The implementation of our method, experimental results, and comparison with other methods are given in Section 5. The conclusion is drawn in Section 6.

2. Density estimation and photon-tracing

The density estimation problem can be stated as follows: given n independent samples $\{X_1, \ldots, X_n\}$ drawn from an unknown probability density function $f(x)$, derive a density function $\hat{f}(x)$ to approximate $f(x)$.

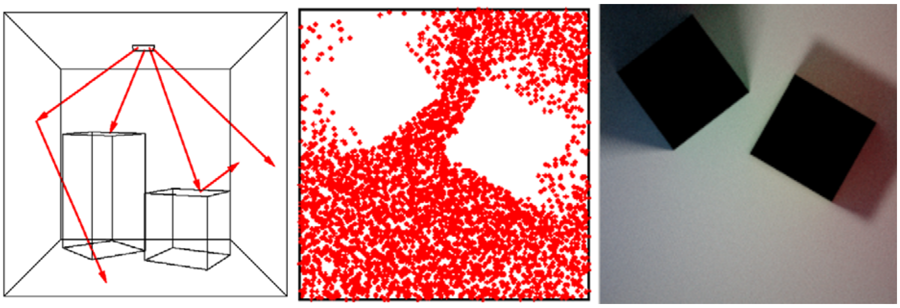

Fig. 1. The photon-tracing process. Left: particles are traced through the scene. Middle: particle-hit points on the floor. Right: estimated illumination of the floor.