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Fundamental limits on end-to-end throughput of network coding in multi-rate and multicast wireless networks

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A B S T R A C T

This paper investigates the interaction between network coding and link-layer transmission rate diversity in multi-hop wireless networks. By appropriately mixing data packets at intermediate nodes, network coding allows a single multicast flow to achieve higher throughput to a set of receivers. Broadcast applications can also exploit link-layer rate diversity, whereby individual nodes can transmit at faster rates at the expense of corresponding smaller coverage area. We first demonstrate how combining rate-diversity with network coding can provide a larger capacity for data dissemination of a single multicast flow, and how consideration of rate diversity is critical for maximizing system throughput. Next we address the following question: given a specific topology of wireless nodes, what is the maximum rate that can be supported by the resultant network exploiting both network coding and multi-rate? We present a linear programming model to compute the maximal throughput that a multicast application can achieve with network coding in a rate-diverse wireless network. We also present analytical results where we observe noticeably better throughput than traditional routing. This suggests there is opportunity for achieving higher throughput by combining network coding and multi-rate diversity.

1. Introduction

There is an increasing interest in understanding the potential performance gains accruing from the use of network coding in multi-hop wireless environments. In particular, many military battlefield scenarios exhibit two characteristics that appear to motivate the use of network coding: (a) the reliance on bandwidth-constrained, ad hoc wireless links (e.g. using MANETs formed by vehicle-mounted radios in urban insurgencies) and (b) the need to disseminate information (e.g., maps, mission commands) to multiple recipients. The initial results on the power of network coding (**NC**), such as the original demonstration of Ahlswede et al. [1], of how in-network mixing of packets by intermediate nodes helps to achieve a communication capacity that is not achievable solely through routing were obtained for the case of a lossless, wireline network. More recently, several groups have investigated the potential performance gains realized by network coding for both unicast (e.g., [7]) and multicast (e.g., [11]) traffic in wireless environments, for a variety of application scenarios. All of these approaches fundamentally aim to exploit the wireless broadcast advantage (**WBA**) by using, whenever possible, a single link-layer broadcast transmission (of a packet formed by a linear combination of individual packets) to reach multiple neighboring nodes. By saving on the number of independent transmissions needed, network-coding approaches effectively reduce the fraction of time the wireless channel is held by a single transmitting node and thereby help to increase the overall network throughput.

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We believe that there is another degree of freedom in wireless environments, namely *link-layer rate diversity*, that network coding approaches have so far failed to exploit. Most commodity wireless cards are now capable of performing adaptive modulation to vary the link rate in response to the signal-to-interference levels at the receiver. Link rate diversity typically exhibits a *rate-range tradeoff*: if the same transmission power is used for all link transmission rates, then, in general, the faster the transmission rate, the smaller is the transmission range (although, the rate-distance variation in real life is somewhat irregular (e.g., see [2])). While this rate diversity has been extensively exploited for unicast traffic and is often standardized, its use in link-layer broadcasting is relatively limited. For example, while the current IEEE 802.11a/b/g standards mandate the transmission of the control frames (e.g. RTS/CTS/ACK) at the lowest rate (e.g., 6 Mbps for IEEE 802.11a), transmission rates for broadcast data are typically implementation-specific. Recently, however, there has been some work (e.g., [4]) that demonstrates that effective exploitation of such rate diversity by *routing* algorithms for link-layer broadcasts can result in significant (often 6-fold) reduction in the broadcast latency and increase in the achievable throughput.

In this paper, we investigate the impact that the use of such rate-diversity for link layer broadcasts may have on the performance of network coding. In addition, we shall also study the *relative importance* of network coding and link-layer transmission rate diversity. It is easy to conceptualize how the rate-range tradeoff inherent to all link-layer broadcasts might impact the performance of various network coding strategies. Without consideration of rate diversity, network coding algorithms operate on an implicit "more-is-better" assumption: since each broadcast transmission takes the same time, encoding a larger number of packets (for a correspondingly larger set of neighbors) into a single packet *always* results in a more efficient use of the wireless channel. In reality, the existence of the rate-range tradeoff often invalidates this assumption. For example, assume that a node n has a set of packets $\{P_1, P_2, \ldots, P_N\}$ targeted for its neighbors $\{n_1, n_2, \ldots, n_N\}$, where the neighbor indices are arranged in non-increasing order of the link transmission rates. Moreover, let R_i be the link rate between the node-pair (n, n_i). In this case, it is possible that combining the first i packets (transmitted at the rate R_i) proves to be more effective than combining the first $i + 1$ packets, because the additional multiplexing gain achieved is negated by the need to use a disproportional smaller rate R_{i+1} for the packet broadcast. Our goals in this paper are thus to answer the following questions:

(1) If multi-rate diversity and network coding can improve together the throughput of the network.
(2) How does the consideration of transmission-rate diversity affect the maximum throughput that may be achieved by linear network coding in wireless environments, i.e., how sensitive are the achievable throughput curves to the impact of link-rate heterogeneity?
(3) How does the throughput achieved by a combination of rate-diverse transmissions and network coding differ in practice from that achieved by pure routing-based strategies that are rate-diversity aware?

Given the closely-coupled interactions between the degree of encoding, the resultant transmission rate and the contentions on the wireless channel, we focus in this paper on the case of single-source multicast problem. Note that the current paper is not *constructive*, i.e., it does not address the design of specific network-coding algorithms that are better at taking advantage of the rate diversity available in a specific network. Instead, our goal is to understand the fundamental interactions between transmission rate diversity and network coding.

1.1. Contributions of this paper

This paper makes the following contributions towards understanding the basic performance of network-coding for broadcast/multicast applications in wireless environments:

- It demonstrates that network coding and multi-rate diversity can improve together the throughput of the network.
- It provides a linear programming model developed for network coding with fixed rates to the variable rate case multicast.
- It uses this new model to compute the maximal throughput achieved with network coding in a rate-diverse wireless network.
- It presents the fundamental limits on the gain of network coding and multi-rate diversity.
- It presents analytical results showing the gain of network coding with multi-rate diversity.

These results demonstrate that it is possible to exploit link-rate diversity and wireless broadcast advantage to improve network performance. The linear programming model is important to measure how close to the optimal the protocols are and to help in the design of new protocols that exploit network coding and rate diversity.

The rest of the paper is organized as follows. Section 2 describes related work. Section 3 describes an initial result to motivate and establish the interplay between NC and rate diversity. In Section 4 we present the Linear Programming Model that finds the optimal rate the network can achieve. In Section 5, we present analytical results. In Section 6 we present the fundamentals limits on the throughput gain with Network Coding and Multi-rate Diversity. Finally, last section concludes the paper with our main conclusions and an enumeration of our current research directions.

2. Related work

The research around network coding was motivated by the seminal paper [1], which demonstrated that, in general, the use of in-network encoding of packets could attain an