Optical Fiber Technology 19 (2013) 93–108

Contents lists available at SciVerse ScienceDirect

Optical Fiber Technology

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# Experimental network synchronization via plastic optical fiber

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ARTICLE INFO

*Article history:*
Received 29 February 2012
Revised 11 October 2012
Available online 23 December 2012

*Keywords:*
Network synchronization
Experimental realization
Chaos
Secure optical communication
Chaotic encryption
Chua's circuit

ABSTRACT

In this paper, network synchronization of coupled Chua's circuits in star configuration is experimentally studied. In particular, plastic optical fiber (POF) is used in the network like communication channels among chaotic nodes to achieve synchronization. The master signal is sent to multiple slaves through a fiber optical coupler with corresponding electrical/optical and optical/electrical stages. An application to encrypted chaotic communication to transmit analogical signal and image messages to multiple receivers is also given.



## 1. Introduction

In past decades, chaotic synchronization has received a tremendous increasing interest, see e.g. [1–8] and references therein. This property is supposed to have interesting applications in different fields, particularly to design private/secure communication systems, see e.g. [4,5,8–17]; in which the confidential information is encrypted into the transmission chaotic signal by direct modulation, masking, or another technique. At the receiver end, if chaotic synchronization can be achieved, then it is possible to extract the hidden information from the transmitted signal. Chaotic circuits and lasers (see e.g. [18–26]) are ideal candidates for experimental realization in secure communications.

On the other hand, synchronization is required in complex networks with many coupled nodes. Particularly interesting is the scenario where the connected nodes have chaotic behavior. Synchronization in complex dynamical networks has direct applications in different fields, see e.g. [12,27–33]. Promising results on synchronization of coupled chaotic nodes in different topologies are reported in [8,12,27–32,34]. However, synchronization in star coupled networks has direct application in communications, where is possible to transmit information from a single transmitter to multiple receivers. In particular, experimental realization of network synchronization of Chua's circuit like nodes is reported in [29].

The main goals of this paper are: (i) to obtain network synchronization of coupled chaotic Chua's circuits in star topology, considering a single master circuit with four slave circuits via *plastic optical fiber* (*POF*) like communication channels. This objective is achieved by using recent results from complex systems theory. In addition, (ii) to transmit encrypted confidential (analogical and image) messages from a single transmitter to multiple receivers via POF communication channels. Both goals are achieved with experimental realization by using opto-electronics devices. To our knowledge the results have not been reported.

The organization of the paper is as follows: In Section 2, a brief review on network synchronization theory is provided. In Section 3, a mathematical model of Chua's circuit used like nodes is described. Section 4 shows the mathematical model of the star coupled network and its synchronization. While, in Section 5, we show the physical implementation and experimental results for chaos network synchronization using POF as communication channels. In Section 6 we apply the results obtained in the experimental network synchronization to transmit encrypted information from a transmitter to multiple receivers. Finally, some conclusions are given in Section 7.

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http://dx.doi.org/10.1016/j.yofte.2012.11.007

## 2. Brief review on synchronization of complex networks

In this section, we give a brief review on complex dynamical networks, in particular of star coupling topology and its synchronization.

### 2.1. Synchronization of complex network

We consider a *complex network* composes of $N$ identical nodes, linearly and diffusively coupled through the first state of each node. In this network, each node constitutes a *n-dimensional dynamical system*, described as follows

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + u_i, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathbb{R}^n$ are the *state* variables of the node $i$, $u_i = u_{i1} \in \mathbb{R}$ is the *input* signal of the node $i$, and is defined by

$$u_{i1} = c \sum_{j=1}^{N} a_{ij} \Gamma \mathbf{x}_j, \quad i = 1, 2, \ldots, N, \tag{2}$$

the constant $c > 0$ represents the *coupling strength* of the complex network, and $\Gamma \in \mathbb{R}^{n \times n}$ is a constant 0–1 matrix linking coupled state variables. For simplicity, assume that $\Gamma = diag(r_1, r_2, \ldots, r_n)$ is a diagonal matrix with $r_i = 1$ for a particular $i$ and $r_j = 0$ for $j \neq i$. This means that two coupled nodes are linked through their $i - th$ state variables. Whereas, $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{N \times N}$ is the *coupling matrix*, which represents the coupling topology of the complex network. If there is a connection between node $i$ and node $j$, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ for$i \neq j$. The diagonal elements of coupling matrix $\mathbf{A}$ are defined as

$$a_{ii} = -\sum_{j=1, j \neq i}^{N} a_{ij} = -\sum_{j=1, j \neq i}^{N} a_{ji}, \quad i = 1, 2, \ldots, N. \tag{3}$$

If the degree of node $i$ is $d_i$, then $a_{ii} = -d_i$, $i = 1,2,\ldots,N$.

Now, suppose that the complex network is connected without isolated clusters. Then, $\mathbf{A}$ is a symmetric irreducible matrix. In this case, it can be shown that zero is an eigenvalue of $\mathbf{A}$ with multiplicity 1 and all the other eigenvalues of $\mathbf{A}$ are strictly negative [27,28].

Synchronization state of nodes in complex systems, can be characterized by the nonzero eigenvalues of $\mathbf{A}$. The complex network (1) and (2) is said to achieve (asymptotically) **synchronization**, if [28]:

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t), \ \textit{as} \ t \to \infty. \tag{4}$$

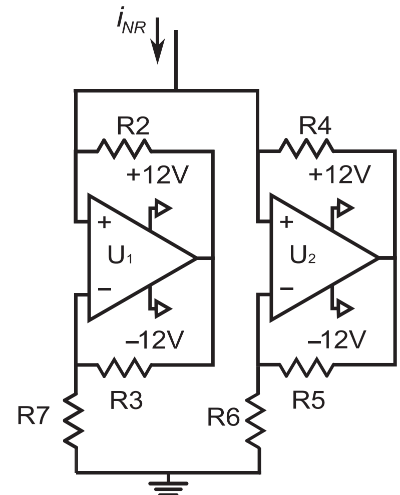


**Fig. 2.** Chua's diode implementation using two Op-Amps and six linear resistors.

The diffusive coupling condition (3) guarantees that the synchronization state is a solution, $\mathbf{s}(t) \in \mathbb{R}^n$, of an isolated node, that is

$$\dot{\mathbf{s}}(t) = f(\mathbf{s}(t)), \tag{5}$$

where $\mathbf{s}(t)$ can be an *equilibrium point*, a *periodic orbit*, or a *chaotic attractor*. Thus, stability of the synchronization state,

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t) = \mathbf{s}(t), \tag{6}$$

of complex network (1) and (2) is determined by the dynamics of an isolated node, i.e. – function $f$ and solution $\mathbf{s}(t)$ – the coupling strength $c$, the inner linking matrix $\Gamma$, and the coupling matrix $\mathbf{A}$.

### 2.2. Synchronization conditions

The following theorem give the conditions to achieve synchronization of the network (1) and (2) as is established in (4).

**Theorem 1** (27,28). *Consider the dynamical network (1) and (2). Let*

$$0 = \lambda_1 > \lambda_2 \geqslant \lambda_3 \geqslant \cdots \geqslant \lambda_N \tag{7}$$

*be the eigenvalues of its coupling matrix A. Suppose that there exists a* $n \times n$ *diagonal matrix* $D > 0$ *and two constants* $\bar{d} < 0$ *and* $\tau > 0$*, such that*

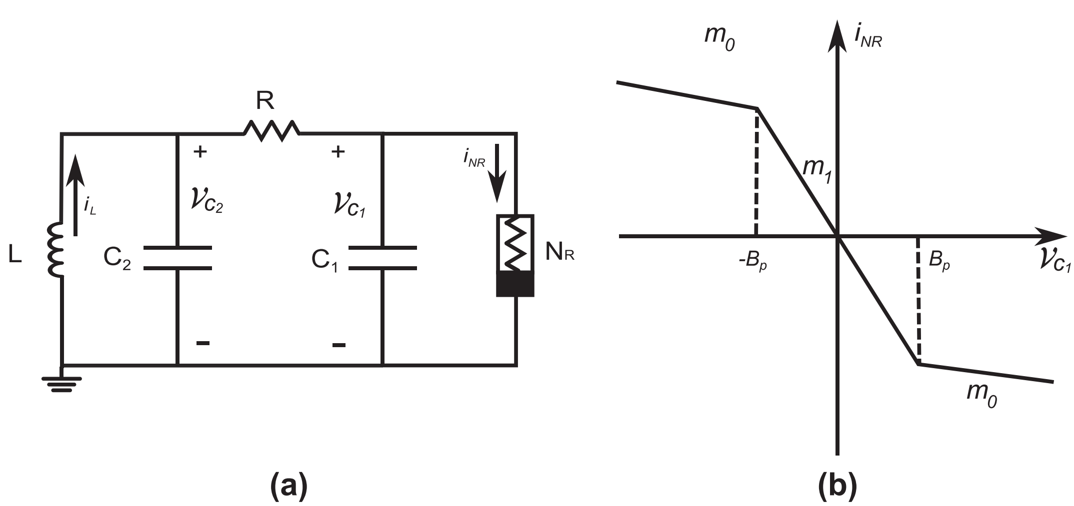


**Fig. 1.** (a) Chua's circuit and (b) $v - i$ characteristic of Chua's diode.