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An application of fractional differintegration to heart rate variability time series

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ABSTRACT

Fractional differintegration is used as a new tool to characterize heart rate variability time series. This paper proposes and focuses in two indexes (α_c and fnQ) derived from the fractional differintegration operator. Both indexes are applied to fractional Gaussian noise (fGn) and actual RR time series in order to test their behavior. In the analysis of monofractal time series, α_c is linearly related with the Hurst exponent and the estimation of the exponent by the proposed index has lower variance than by using Detrended Fluctuation Analysis (DFA) or the periodogram. The other index fnQ quantifies how the time series adjust to a monofractal time series. Age, postural changes and paced breathing cause significant changes on fnQ while α_c only shows significant changes due to posture. In the analyzed actual HRV time series, α_c shows good correlation with the short term scaling exponent obtained by DFA, LF/HF and RMSSD while no correlations have been found for fnQ.

1. Introduction

Heart rate variability (HRV) analysis aims to characterize the variation of the time between consecutive heartbeats or RR interval. HRV is a reliable reflection of many physiological factors modulating the normal rhythm of the heart [1]. The normal variability in heart rate (HR) is due to autonomic neural regulation of the heart and the circulatory system [2]. The balancing action of the sympathetic nervous system (SNS) and parasympathetic nervous system (PNS) branches of the autonomic nervous system (ANS) controls the HR. The degree of variability in the HR provides information about the function of the nervous control on the HR and the heart's ability to react to changes.

In 1996 The European Society of Cardiology and the North American Society of Pacing and Electrophysiology published the results of task force of heart rate variability [3], regarding standards of measurement, physiological interpretation and clinical use. This Task Force defines the three major classes of HRV analysis (time-domain, frequency-domain and nonlinear dynamics), as well as proposes several indexes in the three domains.

The most employed indexes for time-domain analysis with short recordings are the standard deviation of the RR time series (SDNN), regarded as an index of overall variability, and that of the differentiated RR time series (RMSSD) that is used as a surrogate measure of PNS activity. Both indexes work in the same way: the computation of the standard deviation of a certain time series that are related, in this case, by a differentiation operation.

On the other hand, since 1980 but especially from 1990 to now, several works have focused on the fractal-like (or multifractal-like) nature of HRV time series [4] and the

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modeling of RR time series as time series with long-range correlations with a characteristic scaling exponent. Two of the most used processes that show long-range correlations are the fractional Gaussian noise (fGn) and the fractional Brownian motion (fBm). Both processes can be easily obtained from a white Gaussian process after proper fractional differintegration [5] (fractional calculus deals with the generalization of differentiation and integration to non-integer orders and has applications in several fields of science and engineering [6]). Hence, it is surprising the lack of indexes for HRV based on fractional differintegration.

The aim of this work is to present the application of fractional calculus to HRV as a new tool for further development of new indexes. This work will focus on the change with the order of the differintegration operator of the standard deviation of fractionally differintegrated RR time series. The manuscript is organized as follows: first, the differintegration operator and a fast algorithm to perform the operation in time series are introduced. Next, the standard deviation of the fractionally integrated RR time series for a fractional differintegration of order α (SDFDINN(α)) is defined and two indexes that quantify the evolution of SDFDINN with α are proposed. As we will define, SDNN and RMSSD are SDFDINN(0) and SDFDINN(1) respectively. Finally, the two proposed indexes are tested in some databases in order to ascertain differences due to age, posture and breathing pattern and compared with other HRV indexes.

2. Materials and methods

2.1. *Fractional calculus and the fractional differintegration operator*

There are several definitions for differentiation and integration to an arbitrary order but the most well-known definitions are the Riemann–Liouville and the Grünwald–Letnikov definitions [7]. For discrete time applications, the case of the analysis of the RR time series, the Grünwald–Letnikov definition is the most appropriate and the fractional differintegration operator is defined as:

$$ {}_aD_\alpha^t f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \times \sum_{k=0}^{\left\lfloor \frac{t-a}{h} \right\rfloor} (-1)^k \times \binom{\alpha}{k} \times f(t - k \times h) \tag{1} $$

where a is the initial time, α is the order of the operator, h is the sampling period and

$$ \binom{\alpha}{k} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1) \times \Gamma(\alpha-k+1)} \tag{2} $$

being Γ the gamma function and f the function where the differintegration operator is applied. The order α is positive for fractional differentiation and negative for fractional integration.

For the sake of compatibility with SDNN and RMSSD, in this work we will work directly with the RR time series so by convention the initial time will be $a=0$, $h=1$ beat and equation (1) can be approximated by:

$$ D^\alpha RR(k) \cong \sum_{j=0}^{k} (-1)^j \times \frac{\Gamma(\alpha+1)}{\Gamma(j+1) \times \Gamma(\alpha-j+1)} \times RR(k-j) \tag{3} $$

For $\alpha = 1$ (first order differentiation), equation (3) reduces to:

$$ D^{-1} RR(k) \cong \sum_{j=0}^{k} (-1)^j \times \frac{\Gamma(2)}{\Gamma(j+1) \times \Gamma(2-j)} \times RR(k-j) = RR(k) - RR(k-1) \tag{4} $$

because $\Gamma(2-j) = \infty$ for $j > 1$. On the other hand, for $\alpha = -1$ (first order integration), equation (3) becomes:

$$ D^{-1} RR(k) \cong \sum_{j=0}^{k} (-1)^j \times \frac{\Gamma(0)}{\Gamma(j+1) \times \Gamma(-j)} \times RR(k-j) = \sum_{j=0}^{k} (-1)^j \times \binom{-1}{j} \times RR(k-j) \tag{5} $$

But

$$ (-1)^j \times \binom{j-n-1}{j} = \binom{n}{j} \tag{6} $$

then

$$ (-1)^j \times \binom{-1}{j} = \binom{j}{j} = 1 \tag{7} $$

and finally

$$ D^{-1} RR(k) \cong \sum_{j=0}^{k} RR(k-j) \tag{8} $$

A practical problem with the implementation of equation (3) is the factorial increment of the gamma function as j rises. So a better (and faster) solution is to implement the differintegration operator as an IIR filter. In [7] a direct implementation of this filter is described as:

$$ D^\alpha RR(k) \cong \sum_{j=0}^{k} c_j^\alpha \times RR(k-j) \tag{9} $$

where the coefficients are recursively computed as:

$$ c_0^\alpha = 1, \quad c_j^\alpha = \left(1 - \frac{1+\alpha}{j}\right) \times c_{j-1}^\alpha \tag{10} $$

In this work, all the fractional differintegrations have been performed using expression (9) by previously removing the mean of the RR time series for stability purposes. Fig. 1 shows an example on how an actual RR time series changes with the differintegration order. The application of the operator with a certain α results in a new time series that can be analyzed