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Salt and pepper noise removal based on an approximation of l_0 norm

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ABSTRACT

In this paper, we present a novel variational model for salt and pepper noise removal, and an efficient numerical algorithm for solving it. The proposed model features the use of an approximating function of l_0 norm to measure the closeness of the reconstructed and observed images at the pixels which are not the candidates of the noisy pixels. In addition, the total variation (TV) of the image on the entire image domain is minimized for edge-preserving smoothing. When solving the proposed minimization problem, to reduce the computational complexity from the expression of the approximating function, we use the dual forms of both TV and data terms, and find the solution of the corresponding primal–dual problem. Numerous experiments on real images, and comparisons with TV-L2, TV-L1, and adaptive median filter (AMF) indicate the effectiveness and robustness of the proposed method in salt and pepper noise removal.

1. Introduction

Salt and pepper noise is an impulse type of noise in images. This noise is generally caused by errors in data transmission, failure in memory cell or analog-to-digital converter errors. It takes the form of randomly occurring white and black pixels, which can significantly deteriorate the quality of an image.

For images corrupted by salt and pepper noise, noisy pixels are set alternatively to the minimum or maximum intensity values, giving the image a "salt and pepper" like appearance. On the other hand, unaffected pixels always remain unchanged. We can formulate it to the following mathematical expression

$$f(x) = \begin{cases} s_{min} & \text{with probability } p, \\ s_{max} & \text{with probability } q, \\ u(x) & \text{with probability } 1 - p - q, \end{cases} \tag{1}$$

where f and u are the observed noisy image and the noise-free image, respectively; $[s_{min}, s_{max}]$ is the range of $u(x)$. Without loss of generality, we assume that the intensity value of the image is in the range of $[0, 1]$, thus $s_{min} = 0$ and $s_{max} = 1$; the probability $p + q$ determines the level of the salt and pepper noise.

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There are a large number of works to contribute to salt and pepper noise removal. The classical method is the median filter [1]. The main idea of the median filter is to redefine the intensities of the reconstructed image at each pixel by the median of its neighboring pixels.

This algorithm has been shown to be effective when the noise level is low. However, when the noise level is high, the median filter often results in some degradation in edges and details. To improve the performance of median filter, many methods have been proposed such as the adaptive median filter (AMF) [2], the min–max filter (MMF) [3], the center weighted median filter (CWMF) [4], the progressive switching median filter (PSMF) [5], the tri-state median filter (TSMF) [6], the decision-based median filter (DBMF) [7], the iterative median filter (IMF) [8] and so on. These improved filters are good at keeping the uncorrupted intensities, however, all of them replaced the intensities of the noisy pixels by median value of their specific windows, so the edges and details in the restored image still may not be well preserved.

In recent years, the variational method has become more and more popular in image processing. Total variation (TV) minimization, originally introduced in image processing by Rudin, Osher and Fatemi (ROF) in their pioneering work [9], has been widely applied in image restoration and reconstruction [9–19]. A significant advantage of TV regularization is that it leads to an underlying sparse solution (in finite difference domain), and allows for sharp discontinuities in the solution for feature-preserving. The ROF model or TV-L2 model [9] was proposed originally for reducing the Gaussian white noise. It is described as follows

$$\min_u \left\{ \|\nabla u\|_1 + \frac{\lambda}{2}\|u-f\|^2 \right\}, \tag{2}$$

where the first term is the total variation of u, the second term is the data fidelity term measured by l_2 norm, and $\lambda > 0$ is a constant parameter which balances the contributions of the smooth term and the data fitting term.

The TV-L2 model is very good for removing Gaussian noise. But it is not suitable to remove non-Gaussian noise. It has been shown in [20–22] that minimizing the l_1 norm of the difference between the observed and reconstructed images is more appropriate than minimizing l_2 norm of that. The TV-L1 model in [20,21] reads as follows:

$$\min_u \left\{ \|\nabla u\|_1 + \lambda\|u-f\|_1 \right\}, \tag{3}$$

where $\lambda > 0$ is a weighted parameter usually chosen by trial and error method according to the tested images. In [23], the authors observed that the TV-L1 model is "more geometric" in the sense that it makes faster disappearance of the small shapes, such as the salt and pepper noise, than image features with lower contrast. Moreover, the l_1 data fitting leads to a sparse solution that forces the reconstruction error free at many pixels, but allows relatively large error in the certain number of pixels. This is good for removing salt and pepper noise, since the reconstruction error at the noisy pixels should not be very small. However, the TV-L1 model can destroy the uncontaminated intensities especially when the noise level is high because of the difficulty in distinguishing fine structures from the noises.

To avoid the damage of uncontaminated pixels, recently, two-stage algorithms have been introduced to remove impulse noise, and good results have been reported in [24–28]. These methods made use of either the adaptive median filter [25–28] or the center weighted median filter [24,28] to identify the noisy pixels in the first stage, and then, adopt various specialized regularization techniques in the second stage to smooth the pre-selected noisy pixels.

In this paper, we will unify those two steps in a single variational model. As a matter of fact, either the adaptive median filter or the center weighted median filter selects the uncorrupted pixels mainly by eliminating the pixels with maximal and minimal intensities in a window. Based on this observation, we first divide all pixels in the image domain Ω into two disjoint sets: $\tilde{\Omega}$ and $\Omega \setminus \tilde{\Omega}$. The subset $\tilde{\Omega}$ consists of all pixels where the pixel intensities are neither zero nor one (we assume the intensity range is in $[0, 1]$). Hence, there are not any noisy pixels in it, and its complement set $\Omega \setminus \tilde{\Omega}$ contains all noise pixel candidates although it may also have noise-free pixels. At the noise-free pixels we want the reconstructed image u is exactly same as the observed image f, while at the noisy pixels, we do not want u and f close in any sense. Hence, the basic idea of our model is that the intensities of the reconstructed image at the pixels in $\tilde{\Omega}$ should be equal to the observed image, and those at the noisy pixels can be determined using the neighborhood intensity information through an edge-preserving diffusion. Based on this idea, we would like to propose a spatially constrained TV-l_0 model for salt–pepper noise removal.

However, the l_0 norm is a NP-hard problem in terms of computational complexity [29]. In this paper, we introduce a function defined in (7) to approximate the l_0 norm but is easier to compute. We also give the proof of the approximation. We note that the approximating function is closely related to the correntropy and correntropy induced metric (CIM) [30,31] which measure the similarity of two random variables.

Numerically, since the data fitting term in our model involves the approximating function which is nonlinear and non-quadratic, it is difficult to directly optimize. In this work, we adopt the dual forms of both TV and data fitting terms to formulate a corresponding saddle point problem, and then apply the first order primal–dual method to get a solution efficiently.

The rest of this paper is organized as follows. In Section 2, we first define a new function and give an intuitive explanation as well as a theoretical analysis of its approximation to the l_0 norm, and then, propose a novel variational model based on the approximating function for salt and pepper noise reduction. In Section 3, we present our numerical scheme based on the first order primal–dual algorithm. Section 4 shows numerous experimental results and comparisons with several related methods on real images to demonstrate the effectiveness of the proposed method. Finally, we give the conclusion and discuss some future works in Section 5.