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Lie group analysis and numerical solution of magnetohydrodynamic free convective slip flow of micropolar fluid over a moving plate with heat transfer

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ABSTRACT

In this paper, we investigate magnetohydrodynamic free convective flow of micropolar fluid over a moving flat plate using the Lie group transformations and numerical methods. Instead of using conventional no-slip boundary conditions, we used both the velocity and thermal slip boundary conditions to achieve physically realistic and practically useful results. The governing boundary layer equations are non-dimensionalized and transformed into a set of coupled ordinary differential equations (ODEs) using similarity transformations generated by the Lie group, before being solved numerically using Matlab stiff ODE solver *ode15s and* Matlab trust-region-reflective algorithm *lsqnonlin*. The effects of governing parameters on the dimensionless velocity, angular velocity, temperature, skin friction and heat transfer rate are investigated. Our analysis revealed that the dimensionless velocity and angular velocity decrease whilst the dimensionless temperature increases with the velocity slip parameter. Thermal slip reduces the dimensionless velocity and temperature but raises the dimensionless angular velocity. Magnetic field suppresses the velocity but elevates the temperature and angular velocity. Results reported in this paper are in good agreement with the ones reported by the previous authors.

1. Introduction

Magnetohydrodynamics (MHD) of a micropolar fluid is an interesting research area because of its potential applications in different fields of technology. Modern renewable energy systems are increasingly exploiting the MHD technology. The electrically conducting characteristics of many working fluids e.g., ionic solutions, liquid metals, magnetic polymers and others are manipulated by magnetic fields. MHD energy systems are studied in many areas including MHD combustion control [1], MHD duct systems [2], electrolysis MHD designs [3] and entropy generation minimization [4] etc. The study of MHD flow and heat transfer for an electrically conducting fluid are also carried out in electrical power generation, astrophysical flow, solar power technology and space vehicle re-entry [5]. The magnetic field plays as a controlling factor in the convection processes by damping the flow and temperature in material manufacturing fields. In many energy conversion processes, external magnetic fields are used for liquid flows.

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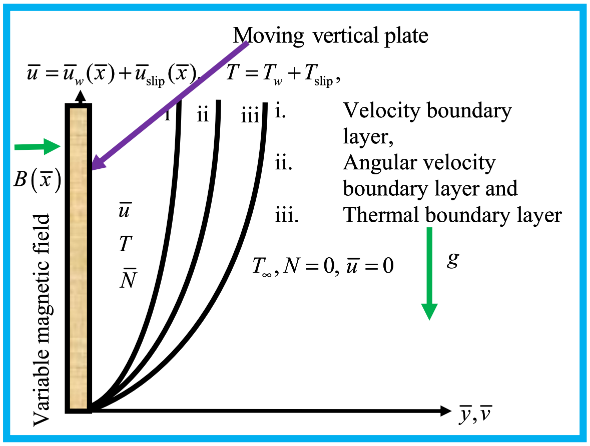

Fig. 1. Flow configuration and coordinate system.

Nowadays the use of micro/nano electromechanical systems (MEMS/NEMS) has been increased in many industries. Such systems have association with the slip flow [6–8]. Several researchers have considered the slip effects on flow and heat transfer. Das et al. [9] investigated the solar radiation effects on cu–water nanofluid flow over a stretching sheet with surface slip and temperature jump. Zhang et al. [10] conducted a review on slip models for gas microflows. Das [11] used Mathematica software to study the slip magnetohydrodynamic radiative reactive flow, heat and mass transfer along an inclined surface. Bég et al. [12] used Maple symbolic methods to analyze the coupled hydromagnetic heat and mass transfer along a moving vertical flat plate with hydrodynamic slip and thermal convective boundary conditions. Uddin et al. [13] analyzed the convective–radiative MHD nanofluid slip flow from a plate, showing that increasing velocity slip and magnetic field boost the nanoparticle concentration, whereas the flow is decelerated as the velocity slip parameter increases.

Micropolar fluid is a fluid containing micro-constituents (e.g., rigid macromolecules) that can undergo rotation. Liquid crystals, colloidal fluids, polymeric suspension and animal blood are some examples of micropolar fluids. The micropolar fluid exhibits certain microscopic behavior arising from individual micro-constituent motions influenced by spin inertia of the micro-constituents. Therefore, the flow of a micropolar fluid can be described by hydrodynamics of the granular system in which the spin of each particle and the macroscopic velocity field is coupled. An excellent review of micropolar fluid mechanics was provided by Lukaszewicz [14]. Researchers have conducted research on micropolar fluids due to their diverse engineering applications ranging from biofluid mechanics of blood vessels to sediment transport in rivers. Micropolar fluids have applications in the development of micropolar biomechanical flows. Recently, Rosali et al. [15] analyzed the flow of micropolar fluid past a permeable stretching/shrinking sheet in a porous medium. Engineers and scientists have been engaged in the research of flows of micropolar fluids with different aspects [16–21]. Bhattacharyya et al. [22] reported the radiation effect on the steady flow of micropolar fluid past a shrinking sheet. Mosayebidorcheh [23] reported a two-dimensional micropolar flow in a porous channel with expanding or contracting walls. They used a new hybrid technique based on the differential transform method (DTM) and iterative Newton's method. Aski et al. [24] presented an analysis of micropolar flow in a porous channel using the Adomian Decomposition Method (ADM). Das [25] studied the slip effects on MHD mixed convection stagnation point flow of a micropolar fluid towards a shrinking vertical sheet. Zheng et al. [26] presented the analytical solution of the mixed convective flow of micropolar fluid with slip and buoyancy effects using the homotopy method. Roşca et al. [27] presented dual solutions of boundary layer flow past a permeable shrinking sheet in a micropolar fluid with a second order slip flow model using Matlab.

The aim of this paper is to investigate the effect of the velocity and thermal slips on the MHD free convective flow of micropolar fluid past a moving vertical plate with heat transfer. The flow is modeled using partial differential equations (PDEs) representing the fundamental conservation principles of mass, momentum and energy. Instead of using similarity transformations from the literature directly, we developed these transformations using a Lie group analysis and used the transformations to convert our PDEs into a set of ODEs. Then numerical solutions of the ODEs were obtained using Matlab functions.

2. Modeling of micropolar MHD transport equations

A steady two-dimensional viscous, incompressible micropolar fluid is assumed to flow along a moving vertical plate. The flow model with the coordinate system is displayed in Fig. 1.

We suppose that the constant temperature of the wall is T_w and that of the surrounding fluid is T_∞ and it is assumed that $T_w > T_\infty$ (i.e., plate is heated). A transverse magnetic field of variable strength $B(\bar{x})$ acts normal to the plate. It is also assumed that the fluid flows under slip boundary conditions. The governing transport equations in dimensional form can

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