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A new learning function for Kriging and its applications to solve reliability problems in engineering

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ABSTRACT

In structural reliability, an important challenge is to reduce the number of calling the performance function, especially a finite element model in engineering problem which usually involves complex computer codes and requires time-consuming computations. To solve this problem, one of the metamodels, Kriging is then introduced as a surrogate for the original model. Kriging presents interesting characteristics such as exact interpolation and a local index of uncertainty on the prediction which can be used as an active learning method. In this paper, a new learning function based on information entropy is proposed. The new learning criterion can help select the next point effectively and add it to the design of experiments to update the metamodel. Then it is applied in a new method constructed in this paper which combines Kriging and Line Sampling to estimate the reliability of structures in a more efficient way. In the end, several examples including non-linearity, high dimensionality and engineering problems are performed to demonstrate the efficiency of the methods with the proposed learning function.

1. Introduction

In structural reliability analysis, a fundamental problem is to compute the integral of joint probability density function (PDF) of random variables in the failure domain, i.e. to solve the multifold probability integral defined as [1]

$$P_f = \int_{G(\boldsymbol{x})\leq 0} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \tag{1}$$

where P_f is the failure probability of the structure. $\boldsymbol{x} = [x_1, \ldots, x_n]^T$ is a vector of random input variables such as loads, environmental factors, material properties, and structural geometry and so on. The performance function $G(\boldsymbol{x})$ characterizes the response of the structure and $G(\boldsymbol{x}) \leq 0$ means the structure is a failure at the point $\boldsymbol{x}$. The border between the failure and safe domain is the limit state $G(\boldsymbol{x}) = 0$. $f(\boldsymbol{x})$ denotes the joint PDF of $\boldsymbol{x}$.

It is difficult to evaluate Eq. (1) for general structures directly due to the time-consuming computation with identifying $G(\boldsymbol{x})$ and numerically performing the multi-dimensional integration of $f(\boldsymbol{x})$ over the failure domain. Therefore, various methods have been developed in order to solve the integral, among which Monte Carlo Simulation (MCS) [2,3] is one of the most widely used methods for handling complex problems. However, it is time-demanding for engineering problems. First and second order reliability methods (FORM, SORM) [4,5] approximate the failure probability based on the knowledge of the most probable point (MPP, also known as design point), and require a significantly smaller number of performance

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function evaluations. But in practice they feature the same limitation: the MPP is difficult to search. Then some novel variance reduction techniques, such as Subset Simulation (SS) [6,7], Line Sampling (LS) [8,9] and Importance Sampling (IS) [10,11], have been proposed to address the computational problems. The basic idea of IS is generating points around the MPP in the vicinity of the limit state surface. SS computes the failure probability as a product of conditional probabilities, which can be easily estimated by Markov Chain Monte Carlo (MCMC) simulation [12,13]. LS computes the failure probability based on the optimal important direction which is from the origin of coordinate to the MPP in the standard normal space. These techniques prove to be robust and require much fewer samples than MCS, and LS is especially excellent for high-dimension and low failure probability problems. It is, however, still not practical in applying these methods on the reliability analysis where the finite element models are involved in engineering problems.

As a result, we need a model to substitute the initial expensive model as the performance function in engineering problems. It is called metamodel with various kinds: Quadratic Response Surfaces [14], Polynomial Chaos [15], Kriging [16,17], Neural Network [18], and Support Vector Machine [19,20], etc. In this paper, Kriging is employed. Kriging, developed by Krige [21] and then developed by Matheron [22], is an exact interpolation method and a form of generalized linear regression for the formulation of an optimal estimator in a minimum mean square error sense. Kriging metamodel is constructed by a design of experiments (DoE) and then can provide not only the predicted response at any point, but also the local uncertainty called Kriging variance on the response: the higher the variance, the less certain the prediction. So if a sample point with higher variance is added to the DoE, the Kriging model can get more improvement. This process is called active learning that means the Kriging model is updated by adding new points intelligently. Therefore, active learning functions which determine whether the points are selected to add or not are proposed in these years. Expected Feasibility Function (EFF), the first learning function proposed by Bichon in an efficient global reliability analysis method [16], aims at adding new points with high Kriging variance in the vicinity of the limit state surface. Then a new learning function U is proposed and first applied in the Active learning reliability method combining Kriging and Monte Carlo Simulation (AK-MCS) [23] and later in the method called AK-IS for active learning and Kriging-based IS [24]. Antoine Dumasa et al. [25] proposed a learning criterion, named as PBC based on the distribution of the prediction. This paper proposes a new learning function H that originated from the information entropy theory and Kriging with this learning function can be applied in many reliability methods such as MCS, IS, LS and so on. As LS is a very efficient reliability method, even for high-dimension and low failure probability problems, a new active learning method combining Kriging and LS called AK-LS is constructed to offer more choices for computing the structural reliability.

The paper is organized as follows. Section 2 recalls the basic theory of the Kriging method for the limit state function. In Section 3, a new active learning function H is proposed and verified. Section 4 introduces a method combining LS method and Kriging with the new active learning function. Several numerical and engineering examples including finite-element-based reliability problems are computed to demonstrate the efficiency of the proposed method in Section 5. The paper ends with conclusions in Section 6.

2. Basic theory about Kriging method

Kriging metamodel is an interpolation technique based on statistical theory, which consists of a parametric linear regression model and a nonparametric stochastic process. It needs a design of experiments to define its stochastic parameters and then predictions of the response can be completed on any unknown point. Give an initial design of experiments (initial DoE) $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_{N_0}]$, with $\boldsymbol{x}_i \in R^n$ $(i = 1, 2, \ldots, N_0)$ the ith experiment, and $\boldsymbol{G} = [G(\boldsymbol{x}_1), G(\boldsymbol{x}_2), \ldots, G(\boldsymbol{x}_{N_0})]$, with $G(\boldsymbol{x}_i) \in R$ the corresponding response to $\boldsymbol{X}$. The approximate relationship between any experiment $\boldsymbol{x}$ and the response $G(\boldsymbol{x})$ can be denoted as

$$\hat{G}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{\beta}, \boldsymbol{x}) + \boldsymbol{z}(\boldsymbol{x}) = \boldsymbol{f}^T(\boldsymbol{x})\boldsymbol{\beta} + \boldsymbol{z}(\boldsymbol{x}) \tag{2}$$

where $\boldsymbol{\beta}^T = \left[\beta_1, \ldots, \beta_p\right]$ is a regression coefficient vector. Similar to the polynomial built by response surface method, $\boldsymbol{f}^T(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_p(\boldsymbol{x})]^T$ makes a global simulation in design space. In the ordinary Kriging, $\boldsymbol{F}(\boldsymbol{\beta}, \boldsymbol{x})$ is a scalar and always taken as $\boldsymbol{F}(\boldsymbol{\beta}, \boldsymbol{x}) = \beta$. So the estimated $\hat{G}(\boldsymbol{x})$ can be simplified as

$$\hat{G}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{\beta}, \boldsymbol{x}) + \boldsymbol{z}(\boldsymbol{x}) = \beta + \boldsymbol{z}(\boldsymbol{x}).$$

Here $\boldsymbol{z}(\boldsymbol{x})$ is a stationary Gaussian process [26] and the statistic characteristics can be denoted as

$$E\left(\boldsymbol{z}(\boldsymbol{x})\right) = 0 \tag{3}$$

$$Var\left(\boldsymbol{z}(\boldsymbol{x})\right) = \sigma_z^2 \tag{4}$$

$$Cov[Z(\boldsymbol{x}_i), Z(\boldsymbol{x}_j)] = \sigma_z^2 R(\boldsymbol{x}_i, \boldsymbol{x}_j) \tag{5}$$

where σ_z^2 is the process variance, and $\boldsymbol{x}_i$, $\boldsymbol{x}_j$ are discretional points from the whole samples X. $R(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is the correlation function about $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ with a correlation parameter vector $\boldsymbol{\theta}$. There are several models to define $R(\boldsymbol{x}_i, \boldsymbol{x}_j)$, and the broadly used Gaussian correlative model is selected in the paper which is the best in application and can be formulated by

$$R(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp \sum_{k=1}^{n} \left[-\theta_k \left(\boldsymbol{x}_i^k - \boldsymbol{x}_j^k\right)^{\delta}\right] \quad \theta_k \geq 0,\ 0 \leq \delta \leq 2 \tag{6}$$