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A fitted finite volume method for real option valuation of risks in climate change[☆]

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ABSTRACT

A large number of industries will experience climate change related damages with the climate change processes over the coming years. For example, the risks from sea level rising will be faced. In addition, there are a lot of uncertainties for the climate change in the future. Therefore, making decisions when to invest in the long term the sea level rising risk related projects is important and complex. The complexity of the decisions mainly lies in the evolving nature of the sea level rising risk, particularly due to the global climate change but also the future socio-economic development scenarios.

In this paper, we first regard the sea level and the temperature as the underlying assets, and then develop a real option model to evaluate potential sea level rising risk management opportunities. In the case of American real options, we reformulate the problem to a linear parabolic variational inequality (VI) in two spatial dimensions and develop a power penalty method to solve it. It is shown that the nonlinear partial differential equation (PDE) is uniquely solvable and the solution of the PDE converges to that of the VI at the rate of order $\mathcal{O}(\lambda^{-\frac{k}{2}})$. A so-called fitted finite volume method is proposed to solve the nonlinear PDE in both cases of European and American options, and the convergence of the fully discrete system of equations is obtained. Finally, some numerical experiments are performed to illustrate the theoretical results of this method.

1. Introduction

As climate change progresses over the coming decades, a widening range of industries will suffer from the climate change related damages. The most likely negative effects of climate change will be temperature rise, productivity losses in climate exposed sectors, more intense and more frequent extreme weather events, and so on. According to the latest assessments from the International Energy Agency (IEA), in 2010 greenhouse gases (GHG) emissions increased to the highest carbon emissions in history, and it is hopeless to hold global warming to safe levels [1]. As a result, climatologists have predicted that more and more extreme weather events will have devastating effects on human society and the environment as GHG concentrations in the atmosphere increase.

One of the most obvious influences from climate change is the sea level rising, which is of a common interest to decision makers. Sea level rising causes huge costs for all corporations and governments which are operating or planning to build

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coastal facilities or developments. Moreover, Nicholls and Tol [2] showed that even if mitigation can slow down the speed of sea level rising, the damages cannot be avoided in the twenty first century due to the lagged response of sea level rise to atmospheric temperature rise. However, there is currently a lack of knowledge about the physical processes driving the sea level, since complex models with different time scales are involved, leading to large uncertainty about future sea level values and imposing costs and risks on the society [3–5].

How to measure and manage the risks from climate changes is of great importance. Here we employ real options analysis method to evaluate and deal with the risks, on the basis of which a linear complementarity model is developed. Real options analysis (ROA) is based on the similarity between investment opportunities and financial options. A real option itself, is the right – but not the obligation – to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. Real options analysis deals with the decision making problem by learning more about the uncertainty over time and exercising the option at the most favorable time. Real options are important in strategic and financial analysis because traditional valuation tools such as net present value (NPV) ignore the value of flexibility. Real options allow for the consideration of possible options that are embedded in investment projects, in which the managers have the flexibility to respond to the outcome of uncertainties. As a result, the real options method may accept a project with a negative NPV [6]. With an option to wait, the real options method may delay the execution of the investment activity. The valuation of a real option can be viewed as an investment optimization problem under uncertainty. The idea is to maximize the NPV of the asset incorporating the relevant managerial flexibility, but subject to operational constraints. Therefore, the real options analysis is an effective approach to handle future uncertainties about the sea level rising risk management by providing flexibility in investment decisions [7–14].

Since the value of a real option of American style is determined by a linear complementarity problem, a power penalty approach to the linear complementarity problem is proposed in the present paper. We will approximate the linear complementarity problem by a nonlinear parabolic PDE in two spatial dimensions with a kth power penalty term, and then show that the solution to the nonlinear PDE converges to that of the original complementarity problem at the rate of order $\mathcal{O}(\lambda^{-\frac{k}{2}})$. In addition, a so-called fitted finite volume method is presented for the numerical solution of the two-dimensional nonlinear PDE. This method is based on a finite volume formulation coupled with a fitted approximation technique. The finite volume method possesses a special feature of the local conservativity of the numerical fluxes, and is becoming more and more popular. See, for instance, Wang [15] for degenerate parabolic problems, Leveque [16] for hyperbolic problems, and Liu [17] for elliptic problems.

The paper is organized as follows. In Section 2, a partial differential equation of the real option is established to describe the risks from climate changes, and final and boundary conditions are prescribed. In Section 3, a power penalty approximation is proposed for the original PDE, and its convergence analysis is presented in Section 4. In Section 5, a so-called fitted volume method is proposed for the discretization of the penalized PDE, and error estimates are established. Finally, some numerical experiments are performed to illustrate the theoretical results in Section 6.

2. The PDE real option model

Assume that a sea level rising defense is to be refurbished. Then, there is no further protection if the defense is rebuilt to the existing height in case sea levels increase, and an alternative would be to rebuild the defense wider and higher. However, this is not optimal if sea levels remain unchanged [7,14]. Under the uncertainty, we employ real options analysis method to decide when the investment is made to rebuild the defense wider and higher optimally. To this purpose, we assume that the height of sea level now is X_1 and the height of defense is X_2. We should widen the defense and increase the height in the future when the sea levels reach and exceed the critical value $X^\star$ (optimal exercise price financially or free boundary mathematically), which is to be decided by solving numerically a real option pricing model governed by a partial differential equation. In addition, it is reasonable to presume that $X_1 < X^\star < X_2$.

2.1. The underlying assets

As the sea level and the temperature are two important indices of climate change and there is a close relationship between them, they are chosen to be the underlying assets in our real option model about the defense [7,14]. According to [18,19], the sea level process is a function of the temperature process by assuming the rate of change of the sea level to be proportional to the temperature increase. As pointed out in [18], in fact we suppose that there exists a global mean temperature that would not cause net melting and that the power available for melting is roughly proportional to the deviation from the temperature. Moreover, to show the randomness of future climate change, the Brownian motion is added into the differential equations of sea level and temperature.

Following [20], we assume that the global mean temperature $(Y_t)_{t\geq 0}$ is a one-dimensional Markov process valued in the open subset D with dynamics under the historical measure $\mathbb{P}$ given by

$$dY_t = \theta(\bar{Y}_1(t) - Y_t)dt + \sigma_Y d\hat{W}_Y, \tag{2.1}$$

where $\bar{Y}_1(t)$ is the equilibrium or mean value supported by fundamental and can possibly be a function of time, σ_Y is the volatility caused by shocks, and $\theta > 0$ is the rate by which these shocks dissipate and the variable reverts towards the mean. From the concept of absence of arbitrage opportunities, given the market price of per unit temperature risk λ_Y and