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A comparative study on time-efficient methods to price compound options in the Heston model

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ABSTRACT

The primary purpose of this paper is to provide an in-depth analysis of a number of structurally different methods to numerically evaluate European compound option prices under Heston's stochastic volatility dynamics. Therefore, we first outline several approaches that can be used to price these type of options in the Heston model: a modified sparse grid method, a fractional fast Fourier transform technique, a (semi-)analytical valuation formula using Green's function of logarithmic spot and volatility and a Monte Carlo simulation. Then we compare the methods on a theoretical basis and report on their numerical properties with respect to computational times and accuracy. One key element of our analysis is that the analyzed methods are extended to incorporate piecewise time-dependent model parameters, which allows for a more realistic compound option pricing. The results in the numerical analysis section are important for practitioners in the financial industry to identify under which model prerequisites (for instance, Heston model where Feller condition is fulfilled or not, Heston model with piecewise time-dependent parameters or with stochastic interest rates) it is preferable to use and which of the available numerical methods.

1. Introduction

The compound option goes back to the seminal paper of Black and Scholes [1]. Not only did they derived their famous pricing formulas for vanilla European call and put options, but they also considered how to evaluate the equity of a company that has coupon bonds outstanding. They argued that the equity can be viewed as a "compound option" because the equity "is an option on an option on · · · an option on the firm". It was Geske [2] who first developed a closed-form solution for the price of a vanilla European call on a European call. It turns out that a wide variety of important problems are closely related to the valuation of compound options. Some examples include pricing American puts in [3] and pricing options on portfolios in [4].

It is well known that derivative securities are not well approximated when it is assumed that the underlying assets follow the geometric Brownian motion process proposed by Black and Scholes [1]. In particular, since the payoff of the compound option is a function of the future underlying vanilla option price – and vanilla option prices show in the market that they do not have constant implied volatility – the compound option price itself not only depends on future spot prices, but also on future levels of volatility. There have been numerous efforts to develop alternative asset return models that are capable of capturing the leptokurtic features found in financial market data, and subsequently to use these models to develop option prices that accurately reflect the volatility smiles and skews found in market traded options. One way to develop option

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pricing models that are capable of generating such behavior is to allow the volatility to evolve stochastically, in particular using the square-root process introduced by Heston [5]. In this paper, we incorporate this feature into the price of compound options.

In the case of compound options under geometric Brownian motion dynamics, there exist ''almost'' explicit integral-form solutions. However, in situations involving more general dynamics (such as stochastic volatility), either explicit solutions do not exist or the integrals become difficult to evaluate. In this paper we propose a number of different approaches that can be used to price compound options under [5] stochastic volatility dynamics. A partial differential equation (PDE) approach is implemented using a modified sparse grid (SG) method. This approach provides an efficient and flexible way to compare prices of compound options. More importantly, it is not only restricted to European-type options but can also include American type or other types of exotic options. We also implement a fractional fast Fourier transform technique, a (semi-) analytic valuation formula using Green's function of the logarithmic spot and volatility and a Monte Carlo simulation. Then we compare the methods on a theoretical basis and report on their numerical properties with respect to computational times and accuracy.

The comparison of computational runtimes of the methods is carried out using parallel computing to obtain the highest quality level of the solutions and to reflect the increased use of clusters in the financial engineering sector. The results in our numerical analysis section are important for practitioners in the financial industry, since in a practical environment the application of fast computational approaches is crucial. Therefore, in this situation it is relevant to identify under which model prerequisites (for instance, the Heston model where the Feller condition is fulfilled, the Heston model where the Feller condition is not fulfilled, the Heston model with piecewise time-dependent parameters or the Heston model with stochastic interest rates) it is preferable to use which of the available numerical methods. Our paper provides such comprehensive discussions and so serves a practical purpose.

The remainder of the paper is structured as follows. In Section 2, we outline the framework of the Heston [5] model, based on which we develop and compare a number of different methods to price compound options. A modified sparse grid approach is discussed in Section 3 to numerically evaluate the partial differential equation that describes the mother option prices. In Section 4 we describe a fractional fast Fourier transform technique that makes use of the representation of the compound option price in terms of its exercise probabilities. Thirdly, we study a (semi-)analytic valuation formula for European compound options by means of Green's function in Section 5. A Monte Carlo simulation is considered in Section 6 as an alternative pricing method before the efficiency of these approaches is analyzed through a number of numerical tests in Section 7. Finally, we draw some conclusions in Section 8.

2. The Heston framework and notation

Following the setting in [5], the dynamics for the share price S under the risk neutral measure and its variance υ are governed by the system of stochastic differential equations

$$dS_t = (r-q)S_t dt + \sqrt{\upsilon_t} S dZ_t^1, \tag{1}$$

$$d\upsilon_t = \kappa(\theta - \upsilon_t)dt + \sigma\sqrt{\upsilon_t} dZ_t^2. \tag{2}$$

Here, the variables r and q denote the rate of interest and the dividend yield, respectively. Their difference is the instantaneous drift of the stock price returns under the risk neutral measure. The stochastic process $\{S_t\}_{t\geq 0}$ followed by the stock price is equivalent to a geometric Brownian motion, but its volatility contains an additional source of randomness. Therefore, the process $\{\upsilon_t\}_{t\geq 0}$ represents the instantaneous variance of the spot price with initial variance level $\upsilon_0 > 0$. The positive parameters κ, θ and σ denote the speed of mean-reversion, the level of mean-reversion and the volatility of the variance of the Cox–Ingersoll–Ross process. Additionally, the two Wiener processes $\{Z_t^1\}_{t\geq 0}$ and $\{Z_t^2\}_{t\geq 0}$ are correlated

$$\langle dZ_t^1 dZ_t^2 \rangle = \rho dt,$$

with a constant rate ρ, taking on values between −1 and 1. Within the scope of this model, we investigate various numerical methods for pricing European compound options.

A compound option is an option with another option as its underlying quantity. Due to this nested optionality, the compound option is characterized by two exercise decisions: the intermediate one of the 'mother option' and the final one of the 'daughter option'. When exercising the mother option the holder may decide to receive the underlying option for a fixed strike price paid in advance. That decision depends on the price of the daughter option at that future time and if the right to buy it for a fixed price is advantageous compared to the current market price of the underlying option. In turn, conditional on this prior decision, the second exercise is that of the underlying option at a later point in time. In the following we will deal with a standard compound call option, which is structured out of two European plain vanilla call options. Similarly, one can define a call on put, put on call, and put on put.

Suppose that a compound option expires at some future date T_M with the strike price K_M and the daughter option on which it is contingent, expires at a later time $T_D(> T_M)$ with the strike price K_D. Under geometric Brownian motion dynamics, the price of such a European call on a European call $M(S,t)$ (or call on call in short) at time t may be expressed as

$$M(S,t) = \mathbb{E}^{\mathbb{Q}}\left[e^{-r(T_M-t)}(D_{BS}(S_{T_M}, T_M) - K_M)^+ | S_t = S\right], \tag{3}$$