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Semi-analytical response of acoustic logging measurements in frequency domain

Ignacio Muga [a,*], David Pardo [b,c,d], Paweł J. Matuszyk [e,1], Carlos Torres-Verdín [e]

[a] Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile

[b] Department of Applied Mathematics, Statistics and Operational Research, University of the Basque Country (UPV/EHU), Bilbao, Spain

[c] BCAM (Basque Center for Applied Mathematics), Bilbao, Spain

[d] IKERBASQUE (Basque Foundation for Sciences), Bilbao, Spain

[e] Department of Petroleum & Geosystems Engineering, The University of Texas at Austin, 1 University Station C0300, Austin, TX 78712-0228, USA

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ABSTRACT

This work proposes a semi-analytical method for simulation of the acoustic response of multipole eccentered sources in a fluid-filled borehole. Assuming a geometry that is invariant with respect to the azimuthal and vertical directions, the solution in frequency domain is expressed in terms of a Fourier series and a Fourier integral. The proposed semi-analytical method builds upon the idea of separating singularities from the smooth part of the integrand when performing the inverse Fourier transform. The singular part is treated analytically using existing inversion formulae, while the regular part is treated with a FFT technique. As a result, a simple and effective method that can be used for simulating and understanding the main physical principles occurring in borehole-eccentered sonic measurements is obtained. Numerical results verify the proposed method and illustrate its advantages.

1. Introduction

Sonic logging instruments are routinely used to assess the main mechanical properties of the subsurface, enabling estimation of the amount and type of hydrocarbons present in a reservoir. However, in actual borehole logging conditions, sonic logging measurements are often difficult to interpret, and they may produce a miscalculation in the hydrocarbon estimation. A well-known situation in which sonic measurements are challenging to interpret is when the logging instrument is located at a nonzero distance from the center of the borehole, the so-called *borehole-eccentered* measurements (cf. [1,2]). To facilitate interpretation of borehole-eccentered measurements, it is necessary to develop effective numerical simulation methods in frequency domain, which provide direct information about the dispersion and velocities of the wave propagation phenomena.

The numerical codes developed for this purpose – most of them based on finite-element discretizations – need to be constantly improved and verified. A powerful and simple test is to benchmark these codes with results that can be obtained by a completely different method, the latter being as analytical as possible. Analytical codes are also used to postprocess

* Corresponding author.
E-mail addresses: ignacio.muga@ucv.cl (I. Muga), dzubiaur@gmail.com (D. Pardo), pawel@ices.utexas.edu (P.J. Matuszyk), cverdin@uts.cc.utexas.edu (C. Torres-Verdín).

[1] Current address: Baker Hughes Inc., Houston, USA.

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in real-time acquired sonic waveforms, which enable corrections on the trajectory of the so-called logging-while-drilling (LWD) instruments.

A simple way to obtain analytical representations of solutions is to assume geometric invariance in two of the three spatial dimensions. Here, we assume a concentric borehole model where the vertical and azimuthal directions remain invariant. In this case, the wavefield produced by a (time-harmonic) point source can be represented by an inverse Fourier integral in terms of the axial coordinate, and a Fourier series in terms of the azimuthal coordinate (cf. [3]). The main computational difficulty is to deal with the singularities of the integrand function in the Fourier inversion, which makes impossible a direct application of a FFT technique or any other conventional numerical integration method. Indeed, those singularities are not integrable in the classical (L^1 or Riemann) sense. Therefore, the Fourier inversion only has a meaning in a *distributional sense*. So far, two numerical techniques have been used to compute this last Fourier inversion integral. One technique is known as the *real axis integration* (RAI) method, introduced by Rosenbaum [4] and Tsang & Rader [5]. The other technique employs a deformed integration path in the complex plane and the most common reference to this method is the work of Kurkjian [6].

The RAI method [5,7] is conceptually easy to apply, but it is a method adapted to recover the solution in time-domain rather than in frequency-domain. Basically, by adding a sufficiently large imaginary part in the frequency variable, the singularities of the integrand function are shifted away from the real axis towards the complex plane. This allows to perform a direct integration over the real axis with a FFT technique, at the expense of introducing a solution defined over a complex path of the frequency domain rather than the traditional real frequency domain space. This prevents us from directly studying frequency domain features that are routinely analyzed in borehole acoustics. For that purpose, one would need to perform additional Fourier transforms, which can be cumbersome and prone to numerical errors. Nonetheless, the RAI method can be applied to several logging scenarios (see [8] or [9]).

On the other hand, the method that uses Cauchy theory [6] is well adapted to recover the frequency-domain solution, but it can be difficult to handle because it involves several integrals in the complex plane. Basically, the Fourier inversion integral is computed as the sum of residues given by the singularities, plus integrals over both sides of the brunch cuts where the integrated function is non-analytic. A slight modification to this technique, in which the residues are computed using a Laurent series expansion, can be found in the recent article [10].

In this paper we use a different approach, which is at the same time conceptually simple and adapted to recover the frequency-domain solution. Since the integrand has non-integrable singularities, it must be regarded as *a tempered distribution* (the space of distributions where the Fourier transform is well-defined). Then, by isolating the singularities, we split the integrated function into a non-integrable part (but still a tempered distribution) plus an integrable part in the $L^1(\mathbb{R})$ sense. The non-integrable part is treated analytically using conventional Fourier transform formulae, while the integrable part is treated numerically with a FFT technique over the real axis (no complex integration paths are used). The idea of decomposing the inverse Fourier transform into analytic and numeric counterparts is widely used in many communities, particularly when evaluating Green's functions for applications in electrical engineering, mechanical engineering, geophysics and boundary elements (cf. [11–14]). Our case is particularly challenging because the map of singularities is complicated: we have several of them, each one depends nonlinearly upon the frequency, and most of them have cut-off frequencies.

We apply this approach to compute the (time-harmonic) acoustic response of a fluid-filled borehole due to multipole eccentered excitations, including some extreme situations where the sources are very close to the borehole wall. We verify our technique against the classical results of Kurkjian [6] and the latest finite-element approach proposed in [1].

This paper is organized as follows. First, we describe the model problem to be solved. Then, we derive the mathematical formulation of our semi-analytical method. Finally, we present some benchmarks and challenging numerical results followed by the main conclusions. This paper also contains an Appendix that describes explicitly the radial stress in Fourier domain.

2. Model problem

Using cylindrical coordinates (r, θ, z), we consider a fluid with sound velocity $c_f > 0$ and density $\rho_f > 0$ filling the borehole $\Omega_f := \{(r, \theta, z) : r < R\}$. An isotropic elastic solid of density $\rho_s > 0$ and Lamé coefficients $\lambda, \mu > 0$ is surrounding the borehole in a region $\Omega_s := \{(r, \theta, z) : r > R\}$. The domains are invariant under rotation and translation in the z-direction. We want to analyze the pressure response in the fluid, due to a time-harmonic point source excitation. Single point source is characterized by its angular frequency $\omega > 0$ and placed for convenience at $(r_s, 0, 0)$, where $0 < r_s < R$ (see Fig. 1). More complicated types of sources like eccentered monopoles, dipoles or quadrupoles are obtained by a simple superposition technique (see Section 3.4 or cf. [15]).

The characteristic wavenumbers are denoted by $k_f = \omega/c_f$ for the fluid, and $k_p = \omega(\rho_s/(\lambda + 2\mu))^{1/2}$ together with $k_s = \omega(\rho_s/\mu)^{1/2}$ for the compressional and shear wavenumber of the solid, respectively.

The pressure response in the fluid and the displacement field response in the elastic solid are represented by $p = p(r, \theta, z; \omega)$ and $\boldsymbol{u} = \boldsymbol{u}(r, \theta, z; \omega) = u_r\boldsymbol{e}_r + u_\theta\boldsymbol{e}_\theta + u_z\boldsymbol{e}_z$, respectively, where $\{\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_z\}$ denotes the set of canonical vectors in cylindrical coordinates. The corresponding set of partial derivatives is denoted by $\{\partial_r, \partial_\theta, \partial_z\}$.

Given an amplitude $A \in \mathbb{C}$ of the source, the pressure behavior in fluid domain is governed by the Helmholtz equation:

$$\begin{cases} \Delta p + k_f^2 p = -\dfrac{A}{r}\delta(r - r_s)\delta(\theta)\delta(z) & \text{in } \Omega_f, \\ \partial_r p = \rho_f \omega^2 u_r & \text{over } r = R, \end{cases} \tag{1}$$