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Uncertainty quantification in Discrete Fracture Network models: Stochastic fracture transmissivity

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ABSTRACT

We consider flows in fractured media, described by Discrete Fracture Network (DFN) models. We perform an Uncertainty Quantification analysis, assuming the fractures' transmissivity coefficients to be random variables. Two probability distributions (log-uniform and log-normal) are used within different laws that express the coefficients in terms of a family of independent stochastic variables; truncated Karhunen–Loève expansions provide instances of such laws.

The approximate computation of quantities of interest, such as mean value and variance for outgoing fluxes, is based on a stochastic collocation approach that uses suitable sparse grids in the range of the stochastic variables (whose number defines the stochastic dimension of the problem). This produces a non-intrusive computational method, in which the DFN flow solver is applied as a black-box. A very fast error decay, related to the analytical dependence of the observed quantities upon the stochastic variables, is obtained in the low dimensional cases using isotropic sparse grids; comparisons with Monte Carlo results show a clear gain in efficiency for the proposed method. For increasing dimensions attained via successive truncations of Karhunen–Loève expansions, results are still good although the rates of convergence are progressively reduced. Resorting to suitably tuned anisotropic grids is an effective way to contrast such curse of dimensionality: in the explored range of dimensions, the resulting convergence histories are nearly independent of the dimension.

1. Introduction

Efficient simulation and investigation of subsurface flow is an up-to-date open research topic. The complexity of the problem and the increasing interest of many applications (analysis of pollutant diffusion in aquifers, Oil&Gas enhanced production, nuclear waste geological storage, carbon dioxide geological storage, geothermal applications, energy and gas storage, etc.) make this research issue of great interest. In these applications, the computational domain for the simulations consists of underground geological reservoirs, that usually have huge complex heterogeneous structure and for which only stochastic data are typically available.

Among the models proposed in literature for the simulation of flows in fractured media, we consider here the Discrete Fracture Network (DFN) model [1].

A DFN model describes a geological reservoir as a system of intersecting planar polygons representing the network of fractures in the underground. Fracture intersections are called *traces*. In the present work we consider impervious surround-

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ing rock matrix, so that no flux exchange occurs with the surrounding medium. The quantity of interest is the flow potential, called hydraulic head, given by the sum of pressure and elevation. The hydraulic head is ruled by Darcy's law in each fracture, with additional matching conditions which ensure hydraulic head continuity and flux balance at fracture intersections. Thanks to these matching conditions, the hydraulic head is continuous across traces but jumps of gradients may occur as a consequence of flux exchange between intersecting fractures. Hence, traces are typically interfaces of discontinuities for the gradient of the solution.

Standard finite element methods or mixed finite elements are widely used for obtaining a numerical solution also in this context, but they require mesh elements to conform with the traces in order to correctly describe the irregular behavior of the solution. This poses a severe limitation, since realistic fracture networks are typically very intricate, with fractures intersecting each other with arbitrary orientation, position, density and dimension. A conforming meshing process may result infeasible, or might generate a poor quality mesh, since a coupled meshing process on all the fractures of the system may lead to elongated elements.

In [2–4] the authors propose a PDE-constrained optimization approach to flow simulations on arbitrary DFNs, in which neither fracture/fracture nor fracture/trace mesh conformity is required. The method is based on the minimization of a quadratic functional constrained by the state equations describing the flow on the fractures. The approach totally circumvents the problem of mesh generation, without any need of geometrical modification tailored on the DFN (e.g., fracture or trace removal or displacement). The method has proven to be quite robust on several medium size DFNs [5,6]. Extended Finite Elements (XFEM) [7–9] are used in order to enrich the numerical solution and correctly reproduce irregularities in the solution. For a list of references to other numerical approaches for DFN flow simulations see, e.g., [2,5].

Coming to the topic of the present paper, we observe that since the actual layout and the geophysical properties of fractures in a large-scale geological basin cannot be precisely established in a deterministic way, DFNs are usually built as representations of natural media starting from stochastic distributions derived from "in situ" measurements [1,10]. Obtaining some accurate quantification of the influence of these distributions on the outputs of DFN models is therefore of paramount importance to assess the reliability of the simulation process. Modern techniques of Uncertainty Quantification for PDE-based models allow us to combine accuracy with computational efficiency, a mandatory requirement for our application, where the cost of each single DFN simulation may be by far non-negligible. This motivates our interest in applying UQ techniques, and in particular stochastic collocation methods, to flow simulation in fractured media.

The paper is organized as follows. In Section 2 we describe the DFN model and its numerical discretization. In Section 3 we recall the basic concepts of Uncertainty Quantification that will be used in our analysis, and we establish conditions assuring the analytical dependence of the solution upon the chosen set of independent stochastic variables. Section 4 is devoted to the description of some representative DFNs on which we specify the boundary value problem, assuming randomness in the transmissivity coefficients; the results of some numerical tests are illustrated. Finally, in Section 5 we consider certain Karhunen–Loève (truncated) expansions of the transmissivity coefficients, and we study the effect of increasing the stochastic dimensionality.

2. Model description and numerical discretization

Let us consider a DFN $\mathcal{D}$ given by the union of open planar polygonal sets F_i, with $i = 1, \ldots, I$, called fractures; let us denote by ∂F_i the boundary of F_i and by $\partial\mathcal{D} = \cup_{i=1}^{I} \partial F_i$ the union of all fracture boundaries. We decompose the latter set as $\partial\mathcal{D} = \Gamma_D \cup \Gamma_N$ with Γ_D closed, Γ_N relatively open, and $\Gamma_D \cap \Gamma_N = \emptyset$, $\Gamma_D \neq \emptyset$ being Γ_D the Dirichlet boundary and Γ_N the Neumann boundary. Similarly, the boundary of each fracture is divided in a Dirichlet part $\Gamma_{iD} = \Gamma_D \cap \partial F_i$ and a Neumann part $\Gamma_{iN} = \Gamma_N \cap \partial F_i$, hence $\partial F_i = \Gamma_{iD} \cup \Gamma_{iN}$, with $\Gamma_{iD} \cap \Gamma_{iN} = \emptyset$. For the ease of description we also assume that $\Gamma_{iD} \neq \emptyset$ for all $i = 1, \ldots, I$; this rather strong assumption can be actually relaxed, see Remark 2.1. Boundary data $g_i^D \in \mathrm{H}^{\frac{1}{2}}(\Gamma_{iD})$ and $g_i^N \in \mathrm{H}^{-\frac{1}{2}}(\Gamma_{iN})$ are given and define the Dirichlet and Neumann boundary conditions, respectively, on the boundary ∂F_i. Fractures have arbitrary orientations in space, so $\mathcal{D}$ is a two-dimensional manifold contained in $\mathbb{R}^3$. Traces are denoted by S_m, $m = 1, \ldots, \mathbb{M}$; $\mathcal{S}$ denotes the set of all the traces of the system, and $\mathcal{S}_i$, for $i = 1, \ldots, I$, denotes the subset of $\mathcal{S}$ corresponding to the traces belonging to F_i. We assume that each S_m uniquely identifies a couple of indices $I_{S_m} = \{i, j\}$, such that $S_m \subseteq \bar{F}_i \cap \bar{F}_j$. See Fig. 1 for an example and an illustration of the notation.

According to Darcy's law, the hydraulic head H in $\mathcal{D}$ is determined by a system of equations on each fracture, defined as follows. For the sake of simplicity of notation, in this section we assume that traces are non-intersecting, but we remark that the numerical method described in the following is not affected by this assumption. Let H_i denote the restriction of the solution H to fracture F_i and let $\mathbf{K}_i$ be a symmetric and uniformly positive-definite tensor (the fracture transmissivity). Let us introduce for each fracture the following functional spaces:

$$V_i = \mathrm{H}^1_{0,\mathrm{D}}(F_i) = \left\{ v \in \mathrm{H}^1(F_i) : v_{|\Gamma_{iD}} = 0 \right\},$$

and

$$V_i^D = \mathrm{H}^1_{\mathrm{D}}(F_i) = \left\{ v \in \mathrm{H}^1(F_i) : v_{|\Gamma_{iD}} = g_i^D \right\}.$$