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Benchmarking the immersed finite element method for fluid–structure interaction problems

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ABSTRACT

We present an implementation of a fully variational formulation of an immersed method for fluid–structure interaction problems based on the finite element method. While typical implementation of immersed methods are characterized by the use of approximate Dirac delta distributions, fully variational formulations of the method do not require the use of said distributions. In our implementation the immersed solid is general in the sense that it is not required to have the same mass density and the same viscous response as the surrounding fluid. We assume that the immersed solid can be either viscoelastic of differential type or hyperelastic. Here we focus on the validation of the method via various benchmarks for fluid–structure interaction numerical schemes. This is the first time that the interaction of purely elastic compressible solids and an incompressible fluid is approached via an immersed method allowing a direct comparison with established benchmarks.

1. Introduction

Immersed methods for fluid–structure interaction (FSI) problems were pioneered by Peskin and his co-workers [1,2]. They proposed an approach called the immersed boundary method (IBM), in which the equations governing the fluid motion have body force terms describing the FSI. The equations are integrated via a finite difference (FD) method and the body force terms are computed by modeling the solid body as a network of elastic fibers. As such, this system of forces has singular support (the *boundary* in the method's name) and is implemented via Dirac-δ distributions. The configuration of the fiber network is represented via a discrete set of points whose motion is then related to that of the fluid again via Dirac-δ distributions. We should clarify that the fiber network in question can be configured so as to represent both thin elastic interfaces as well as thick elastic bodies. In the numerical implementation of this method the Dirac-δ distributions are approximated as *functions*. A recent paper by Fai et al. [3] offers a detailed stability analysis in the context of problems with variable density and viscosity.

Immersed methods based on the finite element method (FEM) has been formulated by various authors [4–7]. Boffi and Gastaldi [4] were the first to show that a variational approach to immersed methods does not necessitate the approximation of Dirac-δ distributions as they naturally disappears in the weak formulation. The thrust of the work by Wang and Liu [5] and Zhang et al. [6] was to remove the requirement that the immersed solid be a fiber network. They also included the ability to

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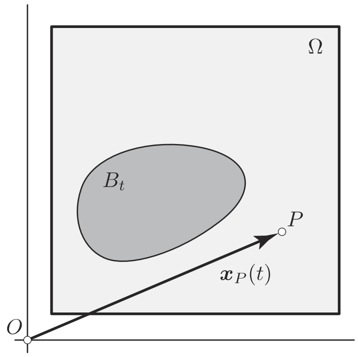

Fig. 1. Current configuration B_t of a body $\mathscr{B}$ immersed in a fluid occupying the domain Ω.

accommodate density differences between solid and fluid, as well as compressible materials in addition to incompressible ones. While they proposed an approach applicable to solid bodies of general topological and constitutive characteristics they maintained the use of approximated Dirac-δ distribution through a strategy called the reproducing kernel particle method (RKPM).

Recently, Heltai and Costanzo [8] proposed a generalization of the approach by Boffi et al. [7] in which a fully variational FEM formulation is shown to be applicable to problems with immersed bodies of general topological and constitutive characteristics and without the use of Dirac-δ distributions. The discussion in [8] focused on the construction of natural interpolation operators between the fluid and the solid discrete spaces that guarantee semi-discrete stability estimates and strong consistency. Since the formulation in [8] is applicable to solid bodies with pure hyperelastic behavior, i.e., without a viscous component in the stress response, in this paper we show that the method in question satisfies the benchmark tests by Turek and Hron [9]. This is an important result given that these benchmarks have become a *de facto* standard in the FSI computational community, and given that previous immersed methods could not satisfy them due to intrinsic model restrictions. In this sense, and to the best of our knowledge, our results are the first of their kind. Along with these important results, we also illustrate the application of our method to a three-dimensional problem whose geometry is similar to that in the two-dimensional benchmark tests.

2. Formulation

2.1. Basic notation and governing equations

Referring to Fig. 1, B_t is a body immersed in a fluid, the latter occupying $\Omega\backslash B_t$, where Ω is a fixed control volume. The body's motion is described by a diffeomorphism $\boldsymbol{\zeta} : B \to B_t, \boldsymbol{x} = \boldsymbol{\zeta}(\boldsymbol{s}, t)$, where B is the body's reference configuration, $\boldsymbol{s} \in B, \boldsymbol{x} \in \Omega$, and time $t \in [0, T]$, with $T > 0$. Away from boundaries, the motion B_t and the fluid are both governed by the balance of mass and momentum, respectively,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \quad \text{and} \quad \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b} = \rho \left[\frac{\partial \boldsymbol{u}}{\partial t} + (\nabla \boldsymbol{u}) \boldsymbol{u} \right], \tag{1}$$

where $\rho(\boldsymbol{x}, t)$ is the mass density, $\boldsymbol{u}(\boldsymbol{x}, t)$ is the velocity, $\boldsymbol{\sigma}(\boldsymbol{x}, t)$ is the Cauchy stress, $\boldsymbol{b}(\boldsymbol{x}, t)$ is the external force density per unit mass, and where ∇ and $(\nabla\cdot)$ denote the gradient and divergence operators, respectively. Eqs. (1) hold both for the solid and the fluid, which can be distinguished via their constitutive equations. We assume that $\boldsymbol{u}(x, t)$ is continuous across ∂B_t, the boundary of B_t. Along with the jump conditions for the momentum balance laws, this implies that the traction field is also continuous across ∂B_t. Eqs. (1) are complemented by the following boundary conditions:

$$\boldsymbol{u}(\boldsymbol{x}, t) = \boldsymbol{u}_g(\boldsymbol{x}, t), \quad \text{for } \boldsymbol{x} \in \partial\Omega_D, \quad \text{and} \quad \boldsymbol{\sigma}(\boldsymbol{x}, t)\boldsymbol{n}(\boldsymbol{x}, t) = \mathfrak{s}_g(\boldsymbol{x}, t), \quad \text{for } \boldsymbol{x} \in \partial\Omega_N, \tag{2}$$

where $\boldsymbol{u}_g$ and $\mathfrak{s}_g$ are prescribed velocity and surface traction fields, $\boldsymbol{n}$ is the outward unit normal to $\partial\Omega$, and where $\partial\Omega_D \cup \partial\Omega_N = \partial\Omega$ and $\partial\Omega_D \cap \partial\Omega_N = \emptyset$.

Fluid's constitutive response. The fluid is assumed to be Newtonian with constant mass density ρ_{f} and stress

$$\boldsymbol{\sigma} = -p\mathsf{I} + \boldsymbol{\sigma}_{\mathrm{f}}^{v} \quad \text{and} \quad \boldsymbol{\sigma}_{\mathrm{f}}^{v} = \mu_{\mathrm{f}}\big(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^{\mathrm{T}}\big), \tag{3}$$

where f stands for 'fluid', p is the pressure, I is the identity tensor, $\boldsymbol{\sigma}_{\mathrm{f}}^{v}$ the linear viscous component of the stress, and $\mu_{\mathrm{f}} > 0$ is the dynamic viscosity. For constant ρ_{f}, the first of Eqs. (1) reduces to $\nabla \cdot \boldsymbol{u} = 0$ ($\boldsymbol{x} \in \Omega\backslash B_t$) and p is a multiplier for the enforcement of this constraint.

Solid's constitutive response. We consider both incompressible and compressible materials. For the incompressible case, Cauchy stress is assumed to be

$$\boldsymbol{\sigma} = -p\mathsf{I} + \boldsymbol{\sigma}_{\mathrm{s}}^{e} + \boldsymbol{\sigma}_{\mathrm{s}}^{v}, \tag{4}$$