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Numerical computation of periodic responses of nonlinear large-scale systems by shooting method

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ABSTRACT

Geometrically nonlinear vibrations of three-dimensional elastic structures, due to harmonic external excitations, are investigated in the frequency domain. The material of the structure is assumed to be linearly elastic. The equation of motion is derived by the conservation of linear momentum in Lagrangian coordinate system and it is discretized into a system of ordinary differential equations by the finite element method. The shooting method is used, to obtain the periodic solutions. A procedure which transforms the initial value problem into a two point boundary value problem, for the periodicity condition, and then it finds the initial conditions which lead to periodic response, is developed and presented, for systems of second order ordinary differential equations. The Elmer software is used for computing the local and global mass and stiffness matrices and the force vector, as well for computing the correction of the initial conditions by the shooting method. Stability of the solutions is studied by the Floquet theory. Sequential continuation method is used to define the prediction for the next point from the frequency response diagram. The main goal of the current work is to investigate and present the potential of the proposed numerical methods for the efficient computation of the frequency response functions of large-scale nonlinear systems, which often result from space discretization of real life engineering applications.

1. Introduction

Very important subject in vibrations of elastic structures is the response of the structure due to external harmonic excitations. Harmonic excitations are fundamental in nature and they have practical applications. Probably the most common tool for analyzing the dynamics of forced vibrations of structures is the frequency response function. It presents the structural response due to applied external force as a function of the excitation frequency. In linear systems, the response due to harmonic excitation is also harmonic function and there exists an analytical expression for the frequency response function. Furthermore, the principle of superposition is valid and it can be used to obtain the total response of the system due to initial conditions and external forces [1].

Nevertheless, majority of the natural systems are nonlinear and linearity is just an approximation. In nonlinear systems, the principle of superposition does not hold, in most cases it is not possible to obtain analytical solution, the response of the systems is not anymore harmonic and it can exist more than one solution. Nonlinearity can significantly change the solution due to a bifurcation point, it can introduce in the system sub-harmonic, super-harmonic, combination or internal resonances, period doubling, quasi periodic or chaotic motions [2]. Stability of the solution is essential for nonlinear systems,

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in contrast to linear ones where all solutions are neutrally stable. Only the stable solutions are of primary interest among the engineers, because the unstable ones do not occur in nature. Stability and bifurcations are related in nonlinear dynamics, because the change in the stability occurs through a bifurcation point [3].

The concepts of nonlinear normal modes (NNM) and nonlinear frequency response functions are related closely. Forced responses occur in the neighborhood of the free oscillations, the shape of vibration of the forced response is identical to that of the neighboring free response, furthermore, appearance of bifurcation point in free vibration, indicates that similar bifurcation can appear in forced vibration [4]. Even though the paper deals with the computation of the nonlinear frequency response function, the definitions of NNM are given for completeness. There are two main definitions of mode of vibration in nonlinear systems, due to Rosenberg [5] and due to Shaw and Pierre [6,7]. Rosenberg defined a nonlinear normal mode of conservative system as motion that all material points vibrate with the same period, achieve their extreme values and static equilibrium positions simultaneously, i.e. the system performs synchronous oscillation (vibration in unison). Shaw and Peirre extended the Rosenberg's definition to damped systems. They defined a nonlinear normal mode as a motion which takes place on a two-dimensional invariant manifold in the phase space, which extends the invariance property of the linear normal modes to nonlinear systems. Recently, Kerschen et al. [8,9], extended the Rosenberg's definition for NNM, to a non-necessarily synchronous but periodic motion. This definition is appropriate in the presence of internal resonances, when the system does not vibrate in unison.

A short literature review of nonlinear vibrations, of discrete and continuous systems, in frequency domain, is presented in the next paragraphs. Vakakis [10] investigated the fundamental and sub-harmonic resonances of two degree-of-freedom system with cubic nonlinearities. The method of multiple scales was used and harmonic excitation was considered. Warminski [11] analyzed the nonlinear dynamics of parametrically excited discrete system of Duffing type. Chaotic motions were found and it was shown that a system under internal resonance condition has higher tendency in transition to chaos.

Lewandowski [12] investigated free vibrations of planar beams by harmonic balance and continuation methods. Ribeiro [13,14] studied the modal interactions of beams due to internal resonance, deriving the equation of motion by using one finite element with hierarchical set of shape functions. Chin and Nayfeh [15] found period doubling and Hopf bifurcations of nonlinear planar responses of hinged–clamped beams due to axial and transverse forces. Non-planar dynamics of beams were investigated, for example in [16–18].

Forced periodic vibrations of rectangular composite plates were investigated in [19]. Chaotic motions were found in forced vibrations of circular plates in [20,21]. Nonlinear free and forced vibrations of shells were investigated, for example, in [22–24]. The equation of motion of shell structure has quadratic plus cubic nonlinearities, due to the curvature of the shell, which can result in softening and/or hardening behavior.

In what concerns three-dimensional structures, discretized by the finite element method, the nonlinear normal modes of full-scale aircraft were computed in [25]. The authors used the shooting method for the periodic solutions plus pseudo-arc length continuation method. Because the resulting system or ordinary differential equations is large, and the nonlinearities of the aircraft were spatially localized, a reduced-order model was applied of the linear part of the system, to reduce the CPU time.

In this work, periodic oscillations of elastic structures are investigated and the solutions are presented in frequency domain. Geometrical type of nonlinearity is considered and the equation of motion is derived by the conservation law of linear momentum in Lagrangian coordinate system. Three-dimensional structures are of interest, thus the partial differential equation is discretized into a system of ordinary differential equations by the finite element method, using three-dimensional elements. Reduced order models, such as beams, plates and shells, are not considered in this work. From one side, the usage of three-dimensional finite elements gives better approximation of the structure and consequently better numerical solution than considering reduced models. On the other side, the usage of three-dimensional elements considerably increases the number of degrees of freedom (DOF) of the system, which requires more computational time and advanced numerical methods for solving the resulting, due to space discretization, algebraic systems. A discussion on the reduction of the CPU time by parallel implementation of the proposed methods is included in the paper.

The shooting method is used for obtaining numerically the initial conditions, which lead to a periodic solution, and the sequential continuation method is used for the prediction of the next solution from the frequency response diagram. The shooting method is derived for systems of second order ordinary differential equations (ODE). This formulation, instead of the common one for systems of first order ODE, is preferred here because of two reasons: it is demonstrated, in the third section of the paper, that the implementation of the shooting method for second order ODE reduces significantly the CPU time; a transformation to first order ODE is not performed, hence additional computations, which result from avoiding the computation of the inverse of the mass matrix, are not performed.

Stability of the periodic solution is determined by the Floquet's multipliers. For that purpose, an explicit expression for the monodromy matrix is defined. It is shown that the monodromy matrix does not need any additional computations and that it results directly from the shooting method.

The proposed numerical methods are validated with a beam model based on the Timoshenko's theory for bending. The periodic responses are compared to results obtained by the harmonic balance method (HBM), applied to the Timoshenko's beam model. It is demonstrated that the results are in agreement, a symmetry-breaking bifurcation point, which was found for the beam model, is also obtained for the three-dimensional beam structure and the secondary branches give also the same periodic oscillations.

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