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A lower bound for the Node, Edge, and Arc Routing Problem

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ABSTRACT

The Node, Edge, and Arc Routing Problem (NEARP) was defined by Prins and Bouchenoua in 2004, although similar problems have been studied before. This problem, also called the Mixed Capacitated General Routing Problem (MCGRP), generalizes the classical Capacitated Vehicle Routing Problem (CVRP), the Capacitated Arc Routing Problem (CARP), and the General Routing Problem. It captures important aspects of real-life routing problems that were not adequately modeled in previous Vehicle Routing Problem (VRP) variants. The authors also proposed a memetic algorithm procedure and defined a set of test instances called the CBMix benchmark. The NEARP definition and investigation contribute to the development of rich VRPs. In this paper we present the first lower bound procedure for the NEARP. It is a further development of lower bounds for the CARP. We also define two novel sets of test instances to complement the CBMix benchmark. The first is based on well-known CARP instances; the second consists of real life cases of newspaper delivery routing. We provide numerical results in the form of lower and best known upper bounds for all instances of all three benchmarks. For three of the instances, the gap between the upper and lower bound is closed. The average gap is 25.1%. As the lower bound procedure is based on a high quality lower bound procedure for the CARP, and there has been limited work on approximate solution methods for the NEARP, we suspect that a main reason for the rather large gaps is the quality of the upper bound. This fact, and the high industrial relevance of the NEARP, should motivate more research on approximate and exact methods for this important problem.

1. Introduction

The Vehicle Routing Problem (VRP) captures the essence of allocation and routing of vehicles at minimal cost, given transportation demand. Hence, it is central to effective and efficient transportation management. VRP research is regarded as one of the great successes of Operations Research, partly due to the emergence of a solution tool industry. Results have been disseminated and exploited in industry. The VRP, construed in a wide sense, is a family of problems. Since the first definition of the classical, Capacitated VRP (CVRP) in 1959 [17], many generalizations have been studied in a systematic fashion. Typically, exact and approximate solution methods have been proposed and investigated for each new VRP variant that has been defined. For an introduction and a survey of the VRP literature, we refer to [35,24].

The VRP is a computationally very hard discrete optimization problem. For industrial cases of reasonable size, one normally has to resort to approximate methods. Efficient procedures for generating proven lower bounds for the optimal value are important both to practice and theory. First, they may speed up exact methods. Second, they provide a benchmark for approximate methods that provide feasible solutions and hence upper bounds on the optimal value. Obviously, a zero gap between an upper and a lower bound for a given instance proves that the value is optimal. A large gap may be due to a poor quality lower bound, a feasible solution of bad quality, or both.

There has been a tremendous increase in the ability to produce exact and approximate solutions to VRP variants over the past 50 years. A few years ago, the best exact methods could consistently solve instances of the CVRP with up to some 70 customers to optimality in reasonable time. Today, the number is above 100, see for instance [7]. Approximate methods such as metaheuristics, matheuristics, and heuristic column generation seem to provide high quality solutions in realistic times even for large-size instances of complex VRP variants. For a categorized bibliography of metaheuristics for the VRP, we refer to [23]. Doerner and Schmid give a survey of matheuristics for VRPs in [18]. In [21], Feillet gives a tutorial on column generation for the VRP.

As problems are regarded as being solved for practical purposes, researchers turn to new extensions and larger-size instances. This trend is enhanced by market pull from the tool industry and their end users. The somewhat imprecise term “rich VRP” has recently been introduced to denote variants that are

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close to capturing all the essential aspects of some subset of real-life routing problems. Generalizations of models in the literature are defined, exact and approximate methods are proposed and investigated, and lower bounds are developed.

In contrast to the CVRP where demand for service is located in the nodes of the network, arc routing problems have been proposed to model the situation where demand is located on edges or arcs in a transportation network [19]. Of particular industrial relevance is the Capacitated Arc Routing Problem (CARP) defined by Golden and Wong in 1981 [25] and its generalizations, as the CARP model contains multiple vehicles with capacity.

There has been a tendency in the literature to dichotomize routing problems into arc routing problems and node routing problems. Some cases are naturally modeled as arc routing because the demand is fundamentally defined on arcs or edges in a transportation network. Prime examples are street sweeping, gritting, and snow clearing. However, the arc routing model has been advocated in the literature for problems where the demand is located in nodes, for instance distribution of subscription newspapers to households and municipal pickup of waste, particularly in urban areas. In real-life cases, there are often thousands or tens of thousands of points to be serviced along a subset of all road links in the area. Such cases are often formulated as CARPs, typically with a drastic reduction of problem size.

In their 2004 paper [34], Prins and Bouchenoua motivate and define the Node, Edge, and Arc Routing Problem (NEARP).[1] They state that:

> Despite the success of metaheuristics for the VRP and the CARP, it is clear that these two problems cannot formalize the requirements of many real-world scenarios.

Their example is urban waste collection, where most demand may adequately be modeled on street segments, but there may also be demand located in points, for instance at supermarkets. Hence, they motivate a generalization of both the classical CVRP and the CARP. To this end, they define the NEARP as a combination of the CVRP and the CARP, which can also be viewed as a capacitated extension of the General Routing Problem [32]. They propose a memetic algorithm for the NEARP and investigate it empirically on standard CVRP and CARP instances from the literature. The authors also create a NEARP benchmark consisting of 23 grid-based test cases, the so-called CBMix-instances, and provide experimental results for their proposed algorithm.

We would like to enhance the motivation for the NEARP and further emphasize its high importance to practice. The arc routing model for node-based demand cases such as subscription newspaper delivery is based on an underlying idea of abstraction. Some form of abstraction may be necessary to contain the computational complexity resulting from a large number of demand points in industrial routing. The assumption that all point-based demands can be aggregated into edges or arcs may be crude in practice. It may lead to solutions that are unnecessarily costly, as partial traversal of edges is not possible. In industry, a route planning task may cover areas that have a mixture of urban, suburban, and rural parts where many demand points will be far apart and aggregation would impose unnecessary constraints on visit sequences. A more sophisticated type of abstraction is aggregation of demand based on the underlying transportation network topology. Such aggregation procedures must also take capacity, time, and travel restrictions into consideration to avoid aggregation that would lead to impractical or low quality plans. In general, such procedure will produce a NEARP instance with a combination of demands on arcs, edges, and nodes. It is therefore imperative to eliminate the arc/node routing dichotomy and thus enable the modeling of the continuum of node and arc routing problems needed for representational adequacy in real-life situations. The introduction of the NEARP was a significant step towards the goal of rich VRP.

Despite its importance, studies of the NEARP are scarce in the literature. The first we know of is the paper by Pandit and Muralidharan from 1995 [33]. They address a generalized version of the NEARP, i.e., routing a heterogeneous fixed fleet of vehicles over specified segments and nodes of a street network, and also include a route duration constraint. The problem is denoted the Capacitated General Routing Problem (CGRP). The authors formally define the CGRP and design a heuristic for solving it. They generate random test instances inspired from curb-side waste collection in residential areas on a network with 50 nodes and 100 arcs. They also investigate the proposed method on random instances of the the Capacitated Chinese Postman Problem for which they had two lower bound procedures.

In [26], the homogeneous fleet specialization of the CGRP studied by Pandit and Muralidharan is investigated by Gutierrez, Soler, and Hervaz. They call the problem the Capacitated General Routing Problem on Mixed Graphs (CGRP-m) and propose a heuristic that compares favorably with the heuristic by Pandit and Muralidharan on the homogeneous fleet case.

Kokubugata et al. [29] study problems from city logistics, including the VRP with Time Windows and the NEARP. They propose a Simulated Annealing metaheuristic for solving these problems. Computational results for the CBMix instances of Prins and Bouchenoua are presented, with several improvements. In [28], Hasle et al. describe results from experiments on NEARP test instances using their industrial VRP solver Spider [27,3], and report new best-known results.

The first integer programming formulation for the NEARP was developed in a forth by Bosco et al. [11]. They extended valid inequalities for the CARP to the NEARP, and embedded them into a branch-and-cut algorithm that was tested on 12 sets of instances constructed from CARP benchmarks. The proposed method could solve only small-size instances, involving at most seven vehicles. Optimal solutions were also provided for two of the smallest CBMix instances.

Lower bounds have been developed for many VRP variants. Many of these are based on cutting planes. See [22,30] for state-of-the-art lower bounds for the CVRP. Also for the General Routing Problem, there is a tradition of obtaining lower bounds through algorithms involving cutting planes. See [14–16] for some of the best lower bound algorithms for this problem.

For the CARP, the academic tradition has been to develop combinatorial lower bounds. Such lower bounds are based on the theory from combinatorial optimization rather than on linear programming. The majority of these bounds are based on the construction of one or several matchings. The best such lower bound is the Multiple Cuts Node Duplication Lower Bound (MCNDLB) [36], with the extensions added in [4]. Good lower bounds based on other strategies are the Hierarchical Relaxations Lower Bound [5], and LP-based bounds [9,31]. Recent exact algorithms using strong lower bounding procedures are found in [8,13]. See [4] for an overview of CARP lower bounds and [37] for a recent survey on CARP in general.

The main contribution of this paper is to provide the first (to the best of our knowledge) lower bound procedure for the NEARP. This bound is inspired by the MCNDLB for CARP and its extensions. We also define two new sets of test instances that complement the grid-based CBMix instances of Prins and Bouchenoua. The first set is called the BHW benchmark. It is based on 20 well-known CARP instances from the literature. The second is called the DI-NEARP benchmark, and consists of 24 instances defined

[1] The NEARP may also be denoted the Mixed Capacitated General Routing Problem.