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# Due-date assignment and machine scheduling in a low machine-rate situation with stochastic processing times

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ABSTRACT

Due date assignment combined with shop floor scheduling has attracted enormous amount of research, in recent years. In many make-to-order situations, the processing times are not known exactly in advance. Further, machine rates are not constant at different times. We assume the case where the machine rate is low at the beginning of the scheduling horizon. However, it can be brought back to normal rate by performing a maintenance activity. The problem includes assigning due-dates and scheduling the jobs and maintenance activity on a single machine where the processing times are stochastic. The objective is minimizing the total cost of lengths of quoted due-dates and expected deviations of completion times from declared due-dates. The optimal solutions of medium-sized problems are found by solving some nonlinear programming models. For larger problems, robust metaheuristics are developed and their performances are statistically analyzed.



## 1. Introduction

Traditionally, the maintenance plan was usually considered as a set of constraints of machine unavailability for shop floor scheduling task. Kacem et al. developed optimal solution approaches minimizing total weighted completion time of machine scheduling problems with a given period of machine unavailability [1]. Ji et al. studied a machine scheduling problem with periodic maintenance minimizing the makespan [2]. They proved that the longest processing time priority rule generates the best approximation of optimal solution.

On the other hand, numerous studies have considered random machine breakdowns in the context of scheduling problems. Liu et al. considered a single machine scheduling problem with dynamic job arrival and multiple random breakdowns [3]. They developed a multi-population genetic algorithm trying to minimize total weighted tardiness and a stability measure. Cai et al. studied a machine scheduling problem with random occurrence of breakdowns and deteriorating jobs and makespan criteria [4]. Tang and Zhao developed a dynamic programming algorithm for machine scheduling minimizing squared deviation of completion times from a common due-date [5]. The studied situation includes both random breakdowns and stochastic processing times.

In the last decade, rate-modifying maintenance activity (maintenance activity, for short) has emerged. This type of maintenance is performed at low-rate processing situations and upgrades the machine processing rate. Maintenance activity has been integrated with machine scheduling problems. Examples include common due-date assignment and maintenance scheduling minimizing total earliness, tardiness, and due-date assignment costs [6] and common due-window assignment and maintenance scheduling with/without deteriorating jobs ([7] and [8], respectively). All of such studies regard that the processing times are deterministic.

From the customer's point of view, order lead-time and on time delivery are among the most important suppliers' competence criteria [9]. This has brought about numerous studies considering the integrated problem of scheduling and due-date quotation. Shabtay [10] considered a machine scheduling minimizing holding, batch delivery, long lead time, and tardiness costs. Li et al. [11] studied machine scheduling in group technology environment minimizing total due-date, earliness, tardiness, and flow time costs. Gordon and Strusevich [12] assumed that the processing times are position-dependent and the objective is minimizing total cost of due-date, earliness, and number of tardy jobs. Shabtay et al. [13] studied several machine scheduling problems with/without controllable processing times and group technology assumption minimizing total cost of due-dates, earliness, tardiness, and resource usage.

The integrated problem of due-date assignment and scheduling becomes more challenging when the processing times are

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stochastic. However, most of the stochastic models either work with completion-time based criteria (e.g. [14]) or assume that the due-dates are exogenous (e.g. [15,16]). A few studies have considered setting due-dates in stochastic environments, some of which are presented as follows.

Portougal and Trietsch [17] studied this problem on a single machine minimizing total expected earliness and tardiness costs. The due-dates were assigned following the given service levels. They proved that some heuristic is asymptotically optimal. Baker and Trietsch [18] studied several problems with due-date cost and service level consideration. For some cases, asymptotically optimal heuristics were developed. Xia et al. [19] worked over the problem with the criterion composed of penalties on earliness, tardiness, and long assigned due-dates. They approximated the objective function and developed a heuristic to find nearly optimal robust solutions. None of these studies consider any type of maintenance.

In the current paper, the integrated problem of due-date quotation and scheduling of jobs and maintenance activity on a single machine is studied. The processing times of jobs are stochastic variables with known mean and variance. In practice, long due-dates can result in sales opportunity loss [10]. On the other hand, declaring unreliable short due-dates can cause tardiness, loss of future sales, or holding costs [10]. The objective of this paper is minimizing total penalties of quoted due-dates and their expected discrepancies with completion times. Clearly, this objective is along with the practical concerns of declaring due-dates which are both short and reliable. The penalty on declared due-dates encourages declaring short lead-times, while the penalty on expected discrepancies precludes quoting unreliable due-dates.

The objective function is non-regular. So, the optimal schedules may include machine idle times. So, either the optimal values of such idle intervals should be calculated (like [20]) or it should be proved that no optimal solution contains machine idle time (like [21]). For the studied problem, it is proved that inserting idle times worsens the objective value of any arbitrary solution.

The paper has the following structure. Section 2 explains the problem formally and introduces the notations. Section 3 derives the optimal terms of due-dates for any specific schedule of jobs and maintenance activity. Section 4 develops an approach to find the global optimal schedules. Except for one special case, finding the optimal solutions seems unlikely in a reasonable time. Hence, in Section 5, two powerful metaheuristics are developed and tuned by a novel robust design method. In Section 6, the computational analysis evaluates both the efficiency and effectiveness of metaheuristics. Finally, conclusion and the interesting areas of future relevant research are overviewed in Section 7.

## 2. Problem definition and notations

This paper studies a machine scheduling problem in which the processing times of jobs are stochastic variables with known values of mean and variance. However, there is no knowledge about their entire distribution. At the beginning of the scheduling horizon, the machine is in a low rate situation and can be brought back to normal state by performing a maintenance activity. Both before and after performing the maintenance, the stochastic variables of processing times are statistically independent. The machine can process one job at a time. However, during the maintenance, the machine is turned off. Moreover, no preemption is allowed.

During the low machine rate, the mean and variance of processing time for job $j$ are $t_j$ and $v_j$, respectively. Performing the maintenance activity enhances the machine rate by factor $\lambda$. The required time to perform the maintenance is a constant denoted by $R$. Let $[j]$ denote the index of job scheduled as the $j$th job. Further, $r$ denotes the position of maintenance activity. Setting maintenance activity at position $r$ means it is scheduled immediately before job $[r]$.

Other parameters are listed here:

| | |
|---|---|
| $n$ | Number of jobs |
| $\gamma$ | Unit cost of promised value of due-date |
| $\theta$ | Unit cost of deviation from promised due-date |
| $E[Y]$ | Expected value of arbitrary stochastic variable $Y$ |
| $V[Y]$ | Variance of arbitrary stochastic variable $Y$ |
| $\sigma[Y]$ | Standard deviation of arbitrary stochastic variable $Y$ |

Further, the decision variables, together with $r$, include

$$x_{ij} = \begin{cases} 1; & \text{if job } i \text{ is scheduled as the } j\text{th job} \\ 0; & \text{otherwise} \end{cases}$$

| | |
|---|---|
| $d_j$ | Due-date assigned to job $j$ |
| $C_j$ | Completion time of job $j$ |

As shown in (1), the objective function is the total due-date assignment cost and expectation of squared bias from due-date.

$$\sum_{j=1}^{n}\left(\gamma d_j + \theta\sqrt{E[(C_j - d_j)^2]}\right) \tag{1}$$

The square root in (1) seems necessary for the sake of dimensional homogeneity. The term under the square root can be written as $E^2[C_j - d_j] + V[C_j - d_j]$. It is similar to the mean-square-error (MSE) which is very common criterion in the literature [22,23].

## 3. Derivation of optimal due-date values

**Theorem 1.** Given the schedule of jobs and maintenance activity, the optimal values of due-dates are calculated through (2).

$$d_j^* = \begin{cases} 0; & \theta \le \gamma \\ E[C_j] - \frac{\gamma}{\sqrt{\theta^2 - \gamma^2}}\sigma[C_j]; & \theta > \gamma \end{cases} \quad j = 1,\ldots,n \tag{2}$$

Proof.

Let $f(d_j)$ be the component of (1) for job $j$ (that is, $f(d_j) = \gamma d_j + \theta\sqrt{E[(C_j - d_j)^2]}$). Clearly, (1) is separable with respect to $d_j$s. So, the optimal values of due-dates can be found, independently. The domain of $f(d_j)$ is $\Re^+ \cup \{0\}$. The first derivative of $f(d_j)$ is shown in (3).

$$\frac{\partial f}{\partial d_j}(d_j) = \gamma + \frac{\theta(d_j - E[C_j])}{\sqrt{d_j{}^2 - 2E[C_j]d_j + E[C_j{}^2]}} \tag{3}$$

Hence, the critical points of $f(d_j)$ are 0 and $\left\{d_j \middle| \frac{\partial f}{\partial d_j}(d_j) = 0\right\}$. The second derivative of $f(d_j)$ is calculated in (4).

$$\frac{\partial^2 f}{\partial d_j^2}(d_j) = \frac{\theta V(C_j)}{\left(d_j^2 - 2E[C_j]d_j + E[C_j^2]\right)^{3/2}} \tag{4}$$

Since expression (4) is positive, $f(d_j)$ is convex. Therefore, $(\partial f/\partial d_j)(d_j) = 0$ has at most one root and either 0 or $\left\{d_j \middle| \frac{\partial f}{\partial d_j}(d_j) = 0\right\}$ is global optimum [24]. The solutions of $(\partial f/\partial d_j)(d_j) = 0$