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A Complex Projection Scheme and Applications

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Abstract—In this paper, an extended projection method in the complex number field is presented. We consider two classes of complex linear matrix inequalities and then derive the corresponding projection operators. Applications to the control system with pole assignment problem and the robust stability of linear descriptor systems, which are described in complex linear matrix inequalities, are given. Based on the numerical algorithms, some examples are illustrated for the merits of the proposed method.

Keywords—Projection method, Linear matrix inequality (LMI), Hermitian matrix, Stability region, Descriptor system.

1. INTRODUCTION

The LMI method has been applied in miscellaneous fields. Due to the explicit formulation and less conservatism, it is involved for solving the concerned control problems recently [1,2]. Thereafter, the control problems, which can be described in LMI manner, are large and continue to grow. However, it is performed well in real number field by some mathematical programming methods, e.g., interior-point polynomial methods proposed in [3]. But for complex LMI, it needs some extra treatment [2].

In this paper, we first formulate two classes of complex LMI and extend the projection method [4,5] to complex number field. For feasible solutions of the described complex LMI, some useful projection operators are further derived. Applications are demonstrated by the pole-assignment problems (e.g., [6–9]) and the robust stability of descriptor systems (e.g., [10–13]) based on the present criteria associated with numerical projection algorithms. The basic idea behind these techniques is that for a class of convex and closed sets, the sequentially alternating projections onto these sets converges to a point in the intersection of the family [14,15]. To

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accelerate the rate of convergence, we can use the direction information towards the intersection [16].

The remainder is organized as follows. In Section 2, the problem formulation is introduced and complex projection operators are derived. The applications to pole-assignment problems and robust stability of descriptor systems are individually demonstrated in Sections 3 and 4, the effectiveness is illustrated by some numerical examples. At length, conclusions are collected in Section 5.

2. PROBLEM FORMULATION AND PROJECTION OPERATORS

Let ℓ_n be the set of Hermitian $n \times n$ matrices equipped with the Frobenius norm $\|X\| = [\mathrm{tr}(X^2)]^{1/2}$ and the inner product $\langle X, Y\rangle = \mathrm{tr}\,\langle XY\rangle$, where $X, Y \in \ell_n$. Let J be a given closed and convex set in ℓ_n and $X^\#$ be a projection onto J from X, then $X^\#$ satisfies $\|X - X^\#\| \leq \|X - \hat{X}\|$ for any matrix $\hat{X}$ in J and $X^\#$ is the unique matrix that satisfies $\langle X^\# - X, X^\# - \hat{X}\rangle \leq 0$ [17,18].

Let $(\cdot)^*$ denotes the complex conjugate transpose of the considered matrix. Consider two classes of linear matrix inequalities formulated by

$$EXF + F^*XE^* \leq -H, \tag{1}$$

$$EXF + F^*XE^* - 2EXE^* \leq -H, \tag{2}$$

where $X \in \ell_n$ is a matrix variable, $H \in \ell_n$ is a constant matrix, and E, F are given complex matrices with compatible dimensions.

Define the following convex sets

$$J_s \triangleq \left\{ W \in \ell_{2n} : [E \quad F^*]\, W \begin{bmatrix} E^* \\ F \end{bmatrix} \leq -H \right\}, \tag{3}$$

$$J_d \triangleq \left\{ W \in \ell_{2n} : W = \begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix},\ X \in \ell_n \right\}, \tag{4}$$

$$J_{\mathrm{de}} \triangleq \left\{ W \in \ell_{2n} : W = \begin{bmatrix} -2X & X \\ X & 0 \end{bmatrix},\ X \in \ell_n \right\}, \tag{5}$$

It is easy to verify that $W \in J_s \cap J_d$ equivalent to (1) and $W \in J_s \cap J_{\mathrm{de}}$ equivalent to (2). Before deriving the projection operators in J_s, J_d, and J_{de}, based on [4], we derive the following result in advance.

LEMMA 1. *Let $X \in \ell_n$ and let $X = LDL^*$ be the eigenvalue-eigenvector decomposition of X, where D is a diagonal matrix of eigenvalues and L is an unitary matrix. The orthogonal projection of X onto the set of negative-semidefinite matrices is given by*

$$X^\# = LD_-L^*, \tag{6}$$

where D_- is the diagonal matrix obtained by replacing the positive eigenvalues of X in D by zeros.

PROOF. Let $\hat{X}$ be a negative-semidefinite matrix, and denote $Y = L^*\hat{X}L$. Since $y_{ii} \leq 0$, then

$$\begin{aligned}
\left\|X - \hat{X}\right\|^2 &= \|LDL^* - LYL^*\|^2 \\
&= \|D - Y\|^2 \\
&= \sum_{i \neq j} |y_{ij}|^2 + \sum_i (\lambda_i - y_{ii})^2 \\
&\geq \sum_{\lambda_i > 0} (\lambda_i - y_{ii})^2 \geq \sum_{\lambda_i > 0} \lambda_i^2 = \left\|X - X^\#\right\|^2,
\end{aligned}$$

where $|\cdot|$ denotes the modulus of the considered complex number. By the definition, $X^\#$ is the orthogonal projection. ∎

The orthogonal projection operators in J_s and J_d sets are directed extended from the results in symmetric matrix [5], and the proofs are thus omitted.