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Algorithms and implementation of a set partitioning approach for modular machining line design

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ABSTRACT

A transfer line design problem is considered. Transfer lines are sequences of workstations equipped with processing modules called blocks each of which performs specific operations. These lines are used for mass production of one type of product and thus execute repetitively a given set of operations. The machine parts move along the stations in the same direction. An identical cost is associated with each station and differing costs are associated with the blocks. The problem is to determine the number of stations, select a set of blocks and assign selected blocks to the stations so that operations of the selected blocks constitute the original set of operations and the total cost is minimized. A distinct feature of the problem is that operations at the same station are performed in parallel. Plus, there are inclusion, exclusion and precedence relations that restrict the assignment of the blocks and operations to the same station as well as the processing order of the operations on the transfer line. We implement a novel set partitioning formulation of this design problem with pre-processing procedures and heuristics. The presented approach has the best performance among the existing methods in terms of solution time and quality.

1. Introduction

The following problem is studied. A transfer line has to be designed for the execution of a given set $N=\{1,\ldots,n\}$ of operations for the mass production of parts of the same type. A set of the required operations is executed on the first station, then another set of the operations on the second station, and so on until each operation of the set N has been executed exactly once. Transport equipment like a conveyor (step-by-step conveyors, transfer-bar-type conveyors, hook conveyors, etc.) is used to move the parts from one station to another in the same direction. Each station can be equipped with a number of processing modules, called *blocks* where each block is allotted a set of operations to perform. Operations of all blocks assigned to the same station are effectuated in parallel. Thus, the processing time for a part at each station is determined by the longest operation of this station. Transfer line cycle time, which is a major characteristic of its performance, is determined by the longest operation of all n operations. If an upper bound on the transfer line cycle time is given, then, without loss of generality, we assume that the time for the longest operation in N does not exceed this upper bound.

The set of available blocks, B, is assumed to be given. Plural *exclusion relations* are imposed on this set. They are represented by a collection E of subsets $E' \subset B$ such that all blocks of E', $E' \in E$, cannot be assigned to the same station. However, any proper subset of E' can be assigned to the same station. These relations inhibit situations where some tools cannot be simultaneously activated on the same station due to a conflict of their physical characteristics, for example, dimensions or blind sides. Assume, without loss of generality, that there are no two sets in E containing one another.

Plural *inclusion* and binary *precedence relations* are given on the set N of operations. Inclusion relations are represented by a collection I of subsets $I' \subset N$ such that all operations of I', $I' \in I$, must be assigned to the same station. These relations model situations where the precision required for some operations can be lost if the part is moved between these operations. Assume without loss of generality that all the sets in I are non-intersecting and that no two blocks from the same exclusion set both contain operations of the same inclusion set.

If operation i *precedes* operation j, which is denoted as $i \to j$, then j cannot be executed on the station of i or any preceding station. Precedence relations reflect technological requirements for operations. Precedence relations are transitive and irreflexive. Assume that they are represented by an acyclic directed graph $G_o=(N,A_o)$, in which there is an arc $(i,j) \in A_o$ if and only if operation i precedes operation j. By this definition, graph G_o coincides with its transitive closure.

Note that the exclusion and inclusion relations restrict the formation of the stations and do not affect the sequence of the

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stations on the transfer line. In contrast, the precedence relations can affect both the station formation and their sequence.

A cost $q(b)>0$ is associated with each block $b \in B$ and a cost $C>0$ is associated with each station. Let $|\cdot|$ denote the cardinality, and let m_0 and n_0 denote upper bounds on the number of stations and blocks for the same station, respectively. In real-life applications, $n_0 \in \{2,3\}$ is often satisfied because, usually, at most two modules are assigned horizontally on both sides of the conveyor and one block can be overhead the conveyor to a station, see such environments in Lapierre et al. [17] and Ozcan and Toklu [19].

The problem is to determine the number of stations, m, sets of blocks assigned to these stations, $W_1,\ldots,W_m$, and sequence of the stations such that $m \le m_0$, $|W_k| \le n_0$, $k=1, \ldots, m$, operations of the assigned blocks constitute set N, exclusion, inclusion and precedence relations are respected, and the total cost,

$$mC+\sum_{k=1}^{m}\sum_{b \in W_k} q(b), \tag{1}$$

is minimized. In the sequel, the notation W_r will be used to designate a station.

This problem will be denoted as problem P. It was first formulated by Dolgui et al. [11] and Belmokhtar et al. [3], who emphasized its practical importance and outlined differences from the assembly line balancing problems and problems of the design of transfer lines with sequential operations. The most recent publications on the latter topics include, among others, Kim et al. [16], Blum and Miralles [5] and Zeng et al. [23].

Problem P was first encountered in the French PCI-SCEMM and Belarusian MZAL companies, which design and manufacture automatic machining lines on the requests of their clients. A client specifies the part to be produced and its production volume (the annual quantity). The line manufacturer designs a process plan and defines nomenclature of tools eligible to perform each operation (eligible blocks). Then it selects blocks, constructs and assembles the machining line for the client. Optimal process planning and equipment selection step is crucial for these machine building companies.

The process planning and equipment selection should take into account the specificity of such lines. Usually, sides of the part are defined according to the part positioning and movement on the line, for example, left, right and top. The required operations are associated with these sides. At each station, a machine part is positioned and several multi-spindle heads (blocks) simultaneously access the part sides, see a sketch of the production environment in Fig. 1. This increases the line's throughput. Several tools are mounted in the same block to perform various operations associated with the same side of the machine part. The tools of the same block are activated simultaneously and may have the same or a different speed. The blocks are either at hand, if the line is reconfigured, or they can be designed and manufactured, if the line is new. Skipping any operation of a block is prohibited, that is, if the block is chosen to be used, then all its operations must be executed. To design a block where some part of which will not be used is economically inefficient.

The line manufacturer would like to select blocks and assign them to the stations so that the set of operations of the selected blocks gives the set of the required operations, total cost is minimized and technological constraints represented by the exclusion, inclusion and precedence relations are satisfied. It may happen that the corresponding problem P has no solution. In this case, the set of available blocks can be extended and problem P can be re-solved. Usually, a block of unit capacity can be designed for each single operation, in which case the set of blocks will not affect the solvability of problem P.

The quality of operations is mostly determined by the sharpness of tools whose control and replacement do not concern this paper. Therefore, this aspect will not be considered.

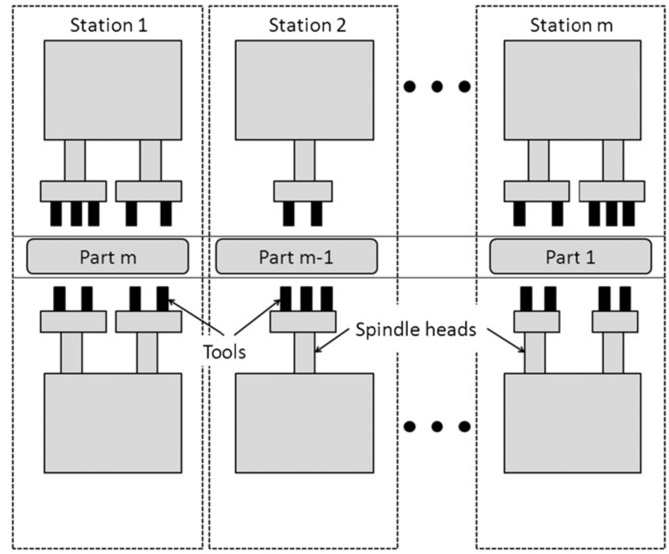

Fig. 1. Production environment.

Dolgui et al. [11] and Belmokhtar et al. [3] suggested two integer programming formulations for problem P and described extensive computer experiments with these formulations using ILOG CPLEX software. Dolgui et al. [9] presented a reduction of problem P to an NP-hard optimal path problem and Delorme et al. [8] to a Maximum Weight Clique problem. Dolgui et al. [10], Guschinskaya and Dolgui [7], and Dolgui and Ihnatsenka [12,13] studied a problem differing from problem P in that operations of different blocks are performed sequentially or in a given order.

In this paper, we suggest a reduction of problem P to a set partitioning problem. Computer experiments with the corresponding software have shown that it outperforms all earlier methods on the benchmark instances.

Set partitioning formulations are used for modeling many real-life optimization problems. For example, Rezanova and Ryan [21] employ such a formulation for workforce scheduling, Rafiee Parsa et al. [20] for scheduling batch processing operations, Vetschera [22] for determining fairness criteria, Garaix et al. [6] for maximizing the passenger occupancy rate in public transportation.

Our reduction and the corresponding mathematical programming model, denoted as MinSPP, are given in Section 2. A key element of the reduction is the concept of *locally feasible station* introduced by Delorme et al. [8]. An outline of the preprocessing procedure is given in Section 3. Depending on the available computational resources, an approximate or exact solution for problem MinSPP is proposed to be constructed. Conventional and randomized greedy heuristics are described in Section 4. A constraint generation algorithm is presented in Section 5, which starts with solving a relaxed version of problem MinSPP, and iteratively adds constraints until the original problem MinSPP is solved or a predetermined time limit is exceeded. An example of problem P and its resolution are given in Section 6. Section 7 describes computer implementation and experiments. The paper concludes with a summary of the results and suggestions for future research.

Preliminary results, which concern ideas of the model and preprocessing, can be found in Borisovsky et al. [4].

2. Reducing P to a set partitioning problem

Our reduction is based on the feature of the actual industrial instances where the number of blocks at the same station usually