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Volume discounting coordinates a supply chain effectively when demand is sensitive to both price and sales effort

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ABSTRACT

In this paper, we use a simple and parsimonious model to investigate the performance of volume discounting schemes (hereafter "[VD]") in a supply chain where the market demand is sensitive to both retail price "p" and sales effort "e" — hereafter called a "(p,e)-channel." The problem is analyzed as a manufacturer-leading Stackelberg game. We first present, for the deterministic-system-parameter situation, contract-designing procedures under two contract formats; namely, a "regular" version of [VD] (hereafter "[RVD]") and a "continuous" version of [VD] (hereafter "[CVD]"). Our solutions show that [RVD] cannot perfectly coordinate this (p,e)-sensitive channel; moreover, very often [RVD] leads to a lower channel efficiency than the simple price-only contract. In contrast, we show that [CVD] leads to perfect channel coordination — a significant result since most contract formats have been shown in the literature to be unable to coordinate a (p,e)-channel. Next, we consider the more realistic situations in which the manufacturer is uncertain about one of the system parameters — specifically, either the market size "a" or the effort cost "η". Our results show that, if Manu is uncertain about a, [RVD] becomes useless but the manufacturer can still use [CVD] to benefit himself. When the manufacturer is uncertain about η, [CVD] remains useful (as expected); however, surprisingly, [RVD] can outperform [CVD] when both the mean value and the uncertainty of η are sufficient high. These results underline the necessity of evaluating a contract format under various forms of system-parameter uncertainties — often at the expense of analytical tractability.

1. Introduction

1.1. Motivation

A major area in supply-chain research is the design of better-performing pricing contracts; i.e., contracts leading to higher channel efficiency and/or higher profit for the contract designer (or the dominant player). The motivation for this research comes from two different directions. First, most earlier studies in this area assume that the product's market demand is set either exogenously or by a single decision variable "p" (the unit retail price). More recently, it is increasingly recognized that "e" (the sales effort level) is a relevant second decision variable. Unfortunately, studies that included "e" as a second demand-setting decision variable have revealed two major gaps:

(a) even in the simpler scenario of no uncertainty in any of the system parameters, a practical contract format that can perfectly coordinate such a channel has yet to be identified; and

(b) the performance of most well-known contract formats deteriorates rapidly as system-parameter uncertainties increase.

As the second direction of motivation, we note that the widely implemented "volume discounting" is one of the earliest pricing contracts to have its channel-coordinating abilities analyzed. However, it has been largely omitted in the earlier contract-design studies for e-dependent demands. Thus, our initial objective was to fill the gap in the literature on "cataloging" the performance of volume discounting under e-dependent demands. As an unexpected but important bonus, we found that a very practical volume-discounting variant is able to perfectly coordinate such a channel when there is no system-parameter uncertainty. Moreover, its performance is very robust against system-parameter uncertainties. These results fill the two major gaps stated in the preceding paragraph.

1.2. Definition of symbols

Manu: name of the male "manufacturer".
Reta: name of the female "retailer".
$\tilde{x}$: A generic random variable, with pdf $f_x(\bullet)$, cdf $F_x(\bullet)$, mean μ_x, standard deviation σ_x, coefficient of variation $\kappa_x \equiv \kappa(\tilde{x}) = \sigma_x/\mu_x$, and finite support $[x_{\min}, x_{\max}]$.
m: The product's unit manufacturing cost incurred by Manu.

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r: The unit processing cost incurred by Reta.
$c \equiv m+r$;
w, p: The product's unit wholesale price and the unit retail price, respectively.
e: Level of sales effort exerted (either by Reta or the "integrated firm").
η: The cost (to Reta or the integrated firm) for exerting sales effort e is ηe^2.
Q: Reta's order quantity.
D: The product's demand.
Π_{Rsub}: Reta's reservation profit or subsistence profit; i.e., Reta will quit the game if her profit is less than Π_{Rsub}.
Π_{X}: Profit of entity "X", where "X" is either "M" (for Manu), "R" (for Reta), "C" (for "Channel", i.e., Manu *plus* Reta), and "I" (for the Integrated firm).
[w]: Label for the Manu-implemented price-only contract.
[RVD]: Label for a "regular" volume discount contract (defined in Section 3.1).
[CVD]: Label for a "continuous" volume discount contract (defined in Section 3.2).
[VD]: Label for generic volume discounting; i.e., [RVD] or [CVD].
$\Pi_{\mathrm{X}}^{[y]}$: Optimal profit of entity "X" achievable under contract format [y] — where [y] may be either [w], [RVD] or [CVD]. Note: "Π_{X}" without a superscript represents merely a profit value at any given situation, but "$\Pi_{\mathrm{X}}^{[y]}$" with a superscript represents the optimal profit under [y].
Θ_{X}: Expected Profit of entity "X"; i.e., $\Theta_{\mathrm{X}} \equiv E(\Pi_{\mathrm{X}})$. It has the same sub/superscript convention as Π_{X}.
CE: Channel efficiency; either $[(\Pi_{\mathrm{M}}+\Pi_{\mathrm{R}})/\Pi_{\mathrm{I}}]$ or $[(\Theta_{\mathrm{M}}+\Theta_{\mathrm{R}})/\Theta_{\mathrm{I}}]$.

Note that variables "w", "e", "p", "Q" and "D" may have the same sub/superscripts as "Π_{X}" defined above.

1.3. Problem statement

Consider a basic manufacturer-Stackelberg channel in which the manufacturer (a male called Manu) sells a product to a retailer (a female called Reta), who in turn retails it to the consumers at \$$p$/unit. We assume the following:

(i) a "(p,e)-channel;" i.e., the retail market demand "D" varies with not only the decision variable p (retail price) but also with the decision variable e (the level of sales effort);

(ii) a linear demand function; i.e.,

$$D(p,e) = a-bp+e; \tag{1}$$

(iii) a quadratic effort-cost function; i.e., the cost of exerting sales effort at level e is ηe^2.
Thus, if Manu imposes a price-only contract [w], the players' profit functions will be:

$$\Pi_{\mathrm{M}} = (w-m)(a-bp+e) \tag{2}$$

$$\Pi_{\mathrm{R}} = [(p-w-r)(a-bp+e)]-\eta e^2 \tag{3}$$

$$\Pi_{\mathrm{I}} = [(p-m-r)(a-bp+e)]-\eta e^2 \tag{4}$$

The formats in (1)–(4) have been used in numerous related studies (e.g., Desai and Srinivasan [1], Taylor [2], Lau et al. [3]).

Keeping in mind that: (i) volume discounting ("[VD]") and many other contract formats (e.g., revenue sharing) are able to coordinate perfectly a manufacturer-Stackelberg channel when demand is sensitive only to p; but (ii) most other contract formats failed to coordinate a (p,e)-channel; our question is: how will [VD] perform? Throughout this paper, we assume that Manu is first interested in maximizing his own profit Π_{M} (or Θ_{M}), then *CE*.

The environmental parameters of the system defined in (1)–(4) are {m, a, b, η, r}. The contract designer in our scenarios (i.e., the dominant Manu) should know his own manufacturing cost "m" well. However, in many situations Manu may have only stochastic knowledge of one or more of {a, b, η, r}, since {a, b} are market parameters, while {η, r} are Reta's internal operational parameters. Thus, besides considering [VD]'s performance when Manu has deterministic knowledge of all the parameters {m, a, b, η, r}, we will consider how [VD]'s performance is affected by uncertainties in "a" and in "η". If Manu perceives (say) a as stochastic, his a-uncertainty level is reflected by a's coefficient of variation $\kappa_a = \sigma_a/\mu_a$.

In our presented numerical examples, we initially assume that Manu's subjective distribution of a is uniform; i.e.,

$$f(a) = 1/(a_{\max}-a_{\min});\, a_{\min} = \mu_a - \sigma_a\sqrt{3}; \quad \text{and} \quad a_{\max} = \mu_a + \sigma_a\sqrt{3} \tag{5}$$

For $\tilde{\eta}$, the symbols κ_η, $f(\eta)$, $\eta_{\max}$ and $\eta_{\min}$ are similarly defined.

1.4. Positioning in the literature

This paper adopts the following convention that differentiates "volume discount" ("[VD]") from "quantity discount" ("[QD]"). [VD] is a discount given on the basis of the total amount ordered over a "period" (typically a year); in contrast, [QD] is a discount given on the basis of the amount ordered in a given batch, regardless of the total volume ordered per year. In one group of discounting studies that include, among others, Lee and Rosenblatt [4], Chakravarty and Martin [5], Hwang et al. [6], Weng [7,8] and Wang [9], situations having substantial logistics costs (i.e., holding, ordering and perhaps shortage costs) are considered; therefore "batch size" and hence quantity discount ([QD]) need to be considered. The product's retail demand in this group may be a constant (as in, e.g., Li and Huang [10], Corbett and de Groote [11], Hofmann [12], Li and Liu [13], Shin and Benton [14], Munson and Hu [15]) or a decreasing function of p (as in, e.g., Abad [16], Weng [7], Viswanathan and Wang [17], Qin et al. [18] and Hsieh et al. [19]). The objective is typically to design a [QD], [VD] or [QD]-cum-[VD] schedule that optimizes either the profit or the "logistics costs," from either Manu's or the channel's perspective.

However, our study is more closely related to a second group of discounting studies, which considers situations where logistics costs are insignificant (e.g., Jeuland and Shugan [20]; Chopra and Meindl [21]; Su and Shi [22]; Burnetas et al. [23]; Lee [24]; Lee and Rhee [25]); hence [QD] becomes irrelevant. Here, one concentrates on studying how "volume"-based [VD] can be used to minimize the negative effect of double marginalization for a product whose retail-market demand is p-sensitive (as in Lau and Lau [26], Lau et al. [27], Yang and Zhou [28], Xiao et al. [29], Zhou [30], Xiao and Qi [31], among many others). Our current study extends this "second group" by considering a retail-market demand that is sensitive to both e and p. It is well known that, in many situations, demand can be increased by various forms of Reta-controlled "sales effort;" e.g., merchandising, point-of-sale or other advertising, improving shelf space, and enhancing sales-personnel service. We use the variable "e" to denote the level of all these various forms of "sales effort". The importance of adding this demand-determining decision variable "e" in supply-chain modeling has been gaining increasing recognition (see, e.g., Lariviere and Padmanabhan [32]; Krishnan et al. [33]; Gurnani and Xu [34]; Xie and Wei [35]; among many others).

A large number of papers have shown that various contract formats (e.g., revenue sharing, resale price maintenance, etc.) can perfectly coordinate a "p-only channel" — i.e., a channel whose