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Constrained location of competitive facilities in the plane

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Abstract

This paper examines a competitive facility location problem occurring in the plane. A new gravity-based utility model is developed, in which the capacity of a facility serves as its measure of attractiveness. A new problem formulation is given, having elastic gravity-based demand, along with capacity, forbidden region, and budget constraints. Two solution algorithms are presented, one based on the big square small square method, and the second based on a penalty function formulation using fixed-point iteration. Computational testing is presented, comparing these two algorithms along with a general-purpose nonlinear solver.

Scope and purpose

In a competitive business environment where products are not distinguishable, facility location plays an important role in an organization's success. This paper examines a firm's problem of selecting the locations in the plane for a set of new facilities such that market capture is maximized across all of the firm's facilities (both new and pre-existing). Customers are assumed to divide their demand among all competing facilities according to a utility function that considers facility attractiveness (measured by facility capacity for satisfying demand) and customer-facility distance. The level of customer demand is assumed to be a function of the facility configuration. Three types of constraints are introduced, involving facility capacity, forbidden regions for new facility location, and a budget function. Two solution algorithms are devised, one based on branch-and-bound methods and the other based on penalty functions. Computational testing is presented, comparing these two algorithms along with a general-purpose nonlinear solver.

Keywords: Competitive facility location; Elastic demand; Penalty functions; Forbidden regions; Nonconvex optimization

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1. Introduction

In the competitive facility location problem, an organization (the “Company”) wishes to open a set of new facilities in an area where pre-existing facilities are operating. Assume that the Company owns a proper subset of the pre-existing facilities. All facilities compete with each other to attract customer demand. The objective is to select the locations for the new facilities that maximize the Company’s total market capture over all of its facilities. Hakimi [1] introduced a more formal problem description. An $(r|X_p)$ *medianoid* is defined as follows: Given a set of p pre-existing facilities that are located at a set of points X_p, where should a competitor locate r new facilities to maximize his market share? This paper generalizes this problem by assuming that the Company may own some of the pre-existing facilities.

A key issue in the modeling of such problems is the representation of demand. It is impractical to consider customers individually, both due to the difficulty of gathering the required information at such a low level, and to the large computational effort required to perform calculations. In practice, customers are aggregated into geographic regions (e.g. three-digit zip codes, municipal boundaries) and all customers in a region are represented by a single demand point. Emir and Francis [2] showed that such aggregation can be performed without introducing much error in the distance calculations. Drezner and Drezner [3] provided a distance correction factor to minimize this error. The authors suggested replacing the distance d to a demand point with $\sqrt{d^2 + \lambda A}$, where A is the region area and λ is a distance correction constant.

Another important consideration is the method for allocating demand to facilities. *Location-allocation models* assume an “all or nothing” approach in that all of the demand at a point is allocated to the most desirable facility. Hotelling [4] published the first modern paper dealing with competitive location. This study dealt with two competitors on a linear segment, with customers patronizing the nearest facility. Hakimi [1] proved that the $(r|X_1)$ medianoid on a network is NP-hard for location-allocation problems. The first to examine these problems in the plane was Drezner [5]. Assuming that all customers patronize the nearest facility, Drezner was able to solve the $(1|X_p)$ medianoid in polynomial time.

Gravity-based models were proposed by Huff [6,7] as an alternative approach to the location-allocation formulation. Huff claimed that the probability of a customer patronizing a facility is directly proportional to the *attractiveness* of the facility (measured as facility size), and is inversely proportional to a power of the customer-facility distance. In general, the utility u_{ij} of demand point i for facility j is defined as the ratio of a nondecreasing function of the attractiveness, and a nondecreasing function of the demand point-facility distance. The probability that a customer at demand point i patronizes facility j is a nondecreasing function of u_{ij}. Note that gravity-based models divide the demand at a given point among all facilities.

Hakimi [1] examined networks where customers follow a gravity model similar to that of Huff. Drezner [8] examined the $(1|X_p)$ medianoid in the plane using the gravity-based model. The solution procedure employed was derived from the Weiszfeld [9] fixed-point iteration procedure. Through computational experience, it was found that many local optimal solutions typically exist. Drezner [10] extended the work of [8] by considering the associated $(r|X_p)$ medianoid. This solution technique was improved by Drezner et al. [11], who used a simulated annealing procedure to find initial solutions, and then utilized a cyclic fixed-point search as in [10] to determine the final facility locations.