Computers & Operations Research 40 (2013) 2724–2739

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Master problem approximations in Dantzig–Wolfe decomposition of variational inequality problems with applications to two energy market models

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ARTICLE INFO

Available online 30 May 2013

Keywords:
Dantzig–Wolfe decomposition
Equilibrium modeling
Variational inequalities
Approximation of the master problem

ABSTRACT

In this paper, a modification to Dantzig–Wolfe (DW) decomposition algorithm for variational inequality (VI) problems is considered to alleviate the computational burden and to facilitate model management and maintenance. As proposals from DW subproblems are accumulated in the DW master problem, the solution time and memory requirements are increasing for the master problem. Approximation of the DW master problem solution significantly reduces the computational effort required to find the equilibrium. The approximate DW algorithm is applied to a time of use pricing model with realistic network constraints for the Ontario electricity market and to a two-region energy model for Canada. In addition to empirical analysis, theoretical results for the convergence of the approximate DW algorithm are presented.

1. Introduction

Decomposition methods sometimes allow large-scale and complex problems to be solved in a distributed and parallel fashion that helps to overcome computational difficulties. They can reduce the memory requirements and/or increase the speed of calculations. Alternatively they can lead to a drastic simplification of the model development procedure and ease the model management and maintenance [27,29]. Generally, the scope of the complex models (e.g., related to public policy making) expands as addressing one question reveals other related questions. Therefore analyses of such models require continuous re-evaluation of the issues. Decomposition of these models allows different analysts or teams of experts to manage, analyze, re-evaluate and repeatedly run sub-models. Expected run time and errors in modeling can be reduced by using decomposition methods [27].

There are several decomposition algorithms (e.g., Dantzig–Wolfe, Benders, Lagrangian) for solving and analyzing large-scale equilibrium problems. Certain models may have a structure that some of the constraints or variables prevent the separability of the problem into subproblems. If these constraints/variables are removed, the resulting subproblems are frequently considerably easier to solve. These constraints/variables are usually referred to as "complicating" (and sometimes referred to as "common" or "linking") constraints/variables [8]. In Dantzig–Wolfe (DW) and Benders decomposition, instead of solving the original problem with complicating constraints or variables, two problems are solved iteratively, a master problem and a subproblem, i.e., original problem without complicating constraints or variables. The solution to the original model is obtained by exchanging price and quantity information among the subproblem(s) and the master problem in an iterative manner. The size of the master problem grows as new solutions (e.g., columns in DW decomposition) from subproblems are passed to master problem and hence, the requirements (e.g., computational time and memory) to solve the master problem increase at each iteration of the decomposition algorithm.

This paper presents modifications to the DW decomposition of variational inequality (VI) problems that allow for the approximation of the master problem to reduce the computational effort required to solve large-scale equilibrium problems and to facilitate the model management and maintenance.

DW decomposition of VI problems has been introduced by Fuller and Chung [16] and Chung et al. [7]. Approximation of the subproblems in DW decomposition of VI problems (single-valued) for decomposition purposes has been presented by Chung and Fuller [6] under useful assumptions. The DW decomposition of VI problems and the approximation of the subproblems have been also studied by Luna et al. [24]. They consider DW decomposition in a more general setting, i.e., for set-valued and maximal monotone VI mappings (in addition to the single-valued, continuous mappings considered by Fuller and Chung [16, Chung et al., [7] and

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http://dx.doi.org/10.1016/j.cor.2013.05.012

Chung and Fuller [6]) as well as various kinds of subproblem approximations that were not considered by Chung and Fuller [6]. Furthermore, Luna et al. [24] consider some algorithmic enhancements, including inexact solution of the approximate subproblem, and the cheap generation of additional proposals by a projection method.

Related to DW decomposition of VI problems, Fuller and Chung [15] also apply Benders decomposition to VI problems and provide convergence results and proofs for a useful class of VI problems. Their algorithm is mainly based on DW decomposition of VI problems and they apply a DW decomposition procedure to a dual of the given VI. By converting the dual forms of DW master and subproblems to their primal forms, they derive the Benders master and subproblems. Gabriel and Fuller [17] apply Benders decomposition to solve a two-stage stochastic complementarity problem (or VI) for an electricity market equilibrium model. Egging [12] also employs Benders decomposition algorithm for large-scale, stochastic multi-period mixed complementarity problems (MCP) for various multi-stage natural gas market models accounting for market power exertion by traders. Because of the primal-dual relations of the DW and Benders' master problems, the approximation for the DW master problem presented in this paper can also be applied to Benders decomposition of VI problems.

In this paper, we firstly introduce the DW decomposition algorithm for VI problems and the approximations for the solution of the master problem in DW decomposition. Convergence analysis is also presented. Numerical investigations are performed on two models in energy markets. These models are a single-period (month) time-of-use (TOU) pricing electricity market equilibrium model with linearized DC network constraints from Çelebi [4] and a realistic two-region energy equilibrium model for Canada from Fuller and Chung [16].

2. Background

VI problems were first developed in the context of studying a class of partial differential equations that arise in the field of mechanics and defined on infinite dimensional spaces [31]. In contrast, finite dimensional VI problems have been studied for computation of economic and game theoretic equilibria. In general, a finite dimensional VI problem is defined as follows:

$$VI(G,K): \quad \text{find a vector } x^* \in K \subseteq R^n, \text{ such that}:$$
$$G(x^*)^T(x-x^*) \geq 0 \quad \forall x \in K \tag{1}$$

where G is a given continuous function from K to R^n, superscript T denotes the transpose, and K is a nonempty, closed and convex set. Standard conditions for existence and uniqueness of solutions to $VI(G,K)$ are provided in Harker and Pang [21], Nagurney [31] or Patriksson [34].

Many mathematical problems (e.g., system of equations, constrained and unconstrained optimization problems, complementarity problems, game theory and saddle point problems, fixed point problems, traffic assignment and network equilibrium problems) can be formulated as VI problems [31,21,2,34]. Unlike an optimization problem which has an objective function, a VI problem has a vector-valued function G, and it is equivalent to an optimization problem only if this vector-valued function is the gradient of an objective function. A necessary and sufficient condition for a differentiable G to satisfy the above condition is that the Jacobian matrix ∇G is symmetric or in other words, that G is integrable, i.e., it can be integrated to define an objective function [31]. Unfortunately, this condition does not hold in many practical problems. In this paper, we consider problems which are non-integrable (asymmetric). See Takayama and Judge [36] and Samuelson [35] for further details on "integrability" conditions.

There are different techniques or algorithms to solve such VI equilibrium models, e.g., by solving a sequence of integrable optimization problems, as in the Project Independence Evaluation System (PIES) algorithm [1], the decoupling algorithm [37], and more general algorithms for VI problems [31]. Alternatively, a VI problem can be converted to an equivalent complementarity problem and solved by Newton methods that solve a sequence of linear complementarity problems [26,25,11,14].

PIES, which was originally developed for energy modeling for US Department of Energy in the 1970s, captures many key features of large-scale equilibrium models. The PIES algorithm approximates the non-integrable equilibrium problem by a sequence of integrable problems which can be converted into equivalent optimization problems. Each iteration solves a linear programming (LP) problem after a proper step function approximation is made on an integrable approximation of the demand function [23]. This algorithm has the characteristics of the nonlinear Jacobi method for solving a system of nonlinear equations. Ahn and Hogan [1] give sufficient conditions under which the PIES algorithm converges. But, as Murphy and Mudrageda [29] point out, although PIES never met these conditions, because of demand function approximations, it usually does not fail to converge.

In our approximations for the solution of the DW master problem, we have also employed the PIES algorithm (as well as another symmetric mapping) to approximate the original mapping in the master problem (see Section 4 for details).

3. Decomposition algorithm and the approximation of the master problem for VI problems

In this section, we summarize the main results of Fuller and Chung [16], using a slightly different notation and following their presentation closely. Then, we present the algorithm with an approximation of the master problem in DW decomposition and its underlying theory of convergence.

3.1. Dantzig–Wolfe decomposition method for VI problems

We consider a VI problem with a feasible set defined by two sets of constraints. We distinguish one of these constraint sets as complicating constraints, e.g., when they are relaxed a VI subproblem is formed (and it may or may not be decomposable, but it is easier to solve or manage). Convex combinations of solutions of the subproblem, together with the complicating constraints, form the feasible set of the master problem. We first define the feasible set for the original VI as follows. All vectors are considered to be column vectors and superscript T denotes the transpose of a vector or matrix. The feasible set is

$$K = \{x \in R^n | g(x) \geq 0,\ h(x) \geq 0\}$$

where g is a mapping from R^n to R^m such that g_i is concave and continuously differentiable for all $i=1,\ldots,m$, and h is a mapping from R^n to R^l such that h_i is concave and continuously differentiable for all $i=1,\ldots,l$. Concavity of g and h ensure convexity of K. The constraints $h(x) \geq 0$ represent the complicating constraints. The vector function G maps R^n to R^n. The original VI is defined as follows:

$$VI(G,K): \quad \text{find } x^* \in K \text{ such that } G(x^*)^T(x-x^*) \geq 0 \quad \forall x \in K \tag{2}$$

We assume throughout this paper that (2) has at least one solution.

The feasible set for the subproblem is defined by relaxing the complicating constraints in K and it is represented as: $\overline{K} = \{x \in R^n | g(x) \geq 0\}$. The subproblem at iteration k is defined with ω^{k-1} (the dual variable vector corresponding to the complicating constraints from the previous master problem solved at iteration