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An adaptive Bayesian scheme for joint monitoring of process mean and variance

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ABSTRACT

This paper presents a new model for the economic optimization of a process operation where two assignable causes may occur, one affecting the mean and the other the variance. The process may thus operate in statistical control, under the effect of either one of the assignable causes or under the effect of both assignable causes. The model employed uses the Bayes theorem to determine the probabilities of operating under the effect of each assignable cause. Based on these probabilities, and following an economic optimization criterion, decisions are made on the necessary actions (stop the process for investigation or not) as well as on the time of the next sampling instance and the size of the next sample. The superiority of the proposed model is estimated by comparing its economic outcome against the outcome of simpler approaches such as Fp (Fixed-parameter) and adaptive Vp (Variable-parameter) Shewhart charts for a number of cases. The numerical investigation indicates that the economic improvement of the new model may be significant.

1. Introduction

In the last decades there has been a massive production of scientific articles proposing numerous economically optimized statistical quality control charts. The pioneering work of Duncan [14], who was the first who associated the use of a simple Shewhart control chart with the economic outcome, and optimized its use, has inspired many scientists for over half a century. An indicative list of early approaches in the field of economically designed control charts would include the works of Bather [2], Goel et al. [17], Knappenberger and Grandage [18], von Collani [40,41] and Duncan [15] who extended his early model to the case of multiple assignable causes. Lorenzen and Vance [19] also proposed an economically designed model that is flexible enough to be used either for Shewhart charts or for Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) charts.

The common characteristic of all aforementioned approaches is that (a) they all assume fixed design parameters and (b) they all consider assignable causes that affect the process mean. Since those early approaches it soon became evident that the design of control charts with adaptive design parameters could significantly improve the economic and statistical behavior of the charts. To this effect, Reynolds et al. [27] were the first to introduce adaptive control charts, and in particular VSI (Variable Sampling Interval) charts. Typical economically designed VSI control charts were later proposed by Das et al. [10] and Bai and Lee [1]. Similar adaptive control charts where the sample size (instead of the sampling interval) is allowed to vary are called VSS (Variable Sample Size) charts and their economic design was first introduced by Park and Reynolds [25]. The economic design of VSSI (Variable Sample Size and sampling Interval) control charts has been analyzed in the works of Das et al. [10] and Park and Reynolds [25] while De Magalhães et al. [12], Costa and Rahim [9], Nenes [21] and Celano et al. [5] present the economic design of fully adaptive control charts (Vp-Variable parameter control charts).

A different stream of economically designed control charts uses the Bayes theorem in order to determine the optimum design parameters. To this end, instead of using just the last sample's outcome, these charts use all information available to reach to the optimum decisions. Bayesian charts have their origins in the first theoretical approaches of Girshick and Rubin [16], Bather [2] and Taylor [38,39]. However, the economic design of Bayesian control charts has not received much attention from the academics, mainly because of the increased modeling complexity. Tagaras [33,34] was the first to introduce a Bayesian one-sided $\bar{X}$ control scheme with adaptive parameters, while, at about the same time, Calabrese [3] developed a one-sided Bayesian *p* chart for monitoring the fraction nonconforming. Porteus and Angelus [26] identify opportunities for cost reduction in SPC through the use of Bayesian

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charts and Tagaras and Nikolaidis [36] compare various one-sided Bayesian $\overline{X}$ control charts from an economic point of view. Nikolaidis et al. [24] present an industrial application of a Bayesian one-sided $\overline{X}$-chart for monitoring the quality of tiles at a particular stage of a tile manufacturing process while Nenes and Tagaras [23], Tagaras and Nenes [35] are the first to introduce a two-sided $\overline{X}$ control scheme for monitoring short production run processes. Makis [20] has introduced an economically designed Bayesian control chart for infinite production processes, keeping the sample size and sampling interval fixed. Celano et al. [4] extend the work of Tagaras [34] and propose an adaptive Bayesian chart for the control of the dispersion of a process while Nenes [22] presents an economically designed fully adaptive Bayesian control chart for monitoring the process mean in infinite production runs.

The economic design of control charts in the presence of assignable causes affecting both the process mean and the process variability has attracted even more limited attention by the scientists because of the increased model complexity. The joint economically optimal design of $\overline{X}$ and R control charts was first considered by Saniga [28] who assumed though, that the occurrence of the one cause blocks the occurrence of the other. Saniga [29] and Saniga and Montgomery [30] developed models for processes subject to a single assignable cause. The occurrence of this assignable cause results in a simultaneous shift in the process mean and variance. Costa [6] presents a model for the joint economic design of $\overline{X}$ and R fixed-parameter control charts where both causes can exist simultaneously. Stoumbos and Reynolds [31] develop a comprehensive economic model for the design of control schemes based on the combination of VSI EWMA and VSI $\overline{X}$ chart schemes. Rahim and Costa [32] deal with the joint economic design of $\overline{X}$ and R charts when the occurrence times of assignable causes follow Weibull distributions with increasing failure rates. Costa [7,8] proposes Joint $\overline{X}$ and R charts with variable sample sizes and sampling intervals (1999) and with all the design parameters adaptive (1998) and investigates numerically the statistical performance of the charts.

De Magalhães and Neto [13] present an economically designed adaptive $\overline{X}$ and R chart. In their paper they assume a single assignable cause which may affect the process mean and/or the standard deviation. However, in their model the design parameters (n, h, w, k) are not allowed to take any possible values, since the optimization procedure necessitates specific rules concerning the relationship between the relaxed and tightened design parameters. De Magalhães et al. [11] propose an adaptive, statistically optimized control chart for monitoring a process subject to two independent assignable causes that affect the process mean and/or the variance. Again, specific rules are utilized that constraint the allowable values of the relaxed and tightened design parameters. Tasias and Nenes [37] also assume two different assignable causes for the mean and standard deviation. However, they follow a different modeling approach allowing a more general problem setting where the two assignable causes are independent and the design parameters of the model can be optimized without any constraints. These same, more generic, assumptions are also made in the present paper, the scope of which is the development of a new Bayesian model that is used to determine the economically optimum parameters in a process where two assignable causes may occur, shifting the mean and/or the variance. Thus, the novel contribution of the paper is twofold:

(a) The development of a model for the representation and economic optimization of a Bayesian chart for monitoring the process mean and variance of a production process when the two assignable causes can occur independently.
(b) The comparison of the economic outcome of the new model against the economic outcome of earlier and less sophisticated approaches.

The remainder of the paper is structured as follows. Section 2 that follows presents in detail the problem setting and assumptions used throughout the paper. Section 3 presents the development of the proposed Bayesian model and describes its operation, while Section 4 describes the expected cost derivation. In Section 5 a numerical investigation is conducted and comparisons with the economic outcome of simpler control charts are presented. Section 6 summarizes the paper and presents its main conclusions.

2. Problem setting and assumptions

A production process is assumed to operate for an infinite horizon of time. The key measure of the process quality is a continuous random variable X which is assumed to be normally distributed. The target mean of X is μ_0 and the target variation is σ_0^2. Occasionally, two assignable causes may affect the process by shifting the mean and variation of the quality characteristic. Assignable cause 1 occurrence is assumed to shift the mean from its target value to $\mu_1 = \mu_0 + \delta\sigma_0$ ($\delta > 0$). In the same sense, assignable cause 2 occurrence shifts the variance from σ_0^2 to $\sigma_1^2 = \gamma^2\sigma_0^2$ ($\gamma > 1$). Note that both assignable causes are assumed to affect the monitored characteristic in a unidirectional way, i.e., only upward shifts are assumed (or only downward shifts for the case of the mean). Unlike many approaches, it is assumed that the occurrence of one cause does not block in any way the occurrence possibility of the other cause. That is, the process, besides operating under statistical control ($\mu = \mu_0, \sigma = \sigma_0$), may operate under the effect of only assignable cause 1 ($\mu = \mu_1, \sigma = \sigma_0$), under the effect of only assignable cause 2 ($\mu = \mu_0, \sigma = \sigma_1$), or under the effect of both assignable causes ($\mu = \mu_1, \sigma = \sigma_1$). The state of the process is denoted by Y=0 when $\mu = \mu_0$ and $\sigma = \sigma_0$, Y=1 when $\mu = \mu_1$ and $\sigma = \sigma_0$, Y=2 when $\mu = \mu_0$ and $\sigma = \sigma_1$ and Y=3 when $\mu = \mu_1$ and $\sigma = \sigma_1$. The notation used throughout this paper is included in Appendix A.

The time until the occurrence of assignable cause 1 is assumed to be an exponentially distributed random variable with mean $1/\lambda_x$ while the time until the occurrence of assignable cause 2 is assumed to be an exponentially distributed random variable with mean $1/\lambda_s$. Thus, the probability of assignable cause 1 occurrence in an interval of h time units is $1-e^{-\lambda_x h}$ and the probability of assignable cause 2 occurrence is $1-e^{-\lambda_s h}$.

It is assumed that both assignable causes are only indirectly observable through the outcome of a sampling procedure. In this sense, samples are collected by the production process and the mean and standard deviation are computed. Based on these computations, a decision may be made not to interrupt the production process ($a = 0$) or an alarm may be issued ($a = 1$) which is followed by an investigation and possible restoration of the production process if any assignable cause has indeed occurred.

The cost of a false alarm is denoted by L_0, the cost of removing assignable cause 1 is denoted by L_x, the cost of removing assignable cause 2 is denoted by L_s, while the cost of removing both assignable causes is denoted by L_{xs}. In the same sense, the cost per time unit of operating under the effect of assignable cause 1 is denoted by M_x, the cost per time unit of operating under the effect of assignable cause 2 is denoted by M_s, while the cost per time unit of operating under the effect of both assignable causes is denoted by M_{xs}. In addition to the aforementioned costs, the fixed cost per sample is denoted by b while the variable sampling cost by c.

The time to interrupt the process and investigate it is denoted by T_0, the time to remove assignable cause 1 is denoted by T_x, the time to remove assignable cause 2 is denoted by T_s while the time to remove both assignable causes is denoted by T_{xs}.