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Variable neighborhood search for location routing

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A R T I C L E I N F O

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A B S T R A C T

In this paper we propose various neighborhood search heuristics (VNS) for solving the location routing problem with multiple capacitated depots and one uncapacitated vehicle per depot. The objective is to find depot locations and to design least cost routes for vehicles. We integrate a variable neighborhood descent as the local search in the general variable neighborhood heuristic framework to solve this problem. We propose five neighborhood structures which are either of routing or location type and use them in both shaking and local search steps. The proposed three VNS methods are tested on benchmark instances and successfully compared with other two state-of-the-art heuristics.

1. Introduction

Location-routing problem. In contrast to location and routing problems, where each decision is considered separately, the location routing problem (LRP) considers both decisions simultaneously which is for many logistic systems, more beneficial. In this paper, we study a variant of the LRP with uncapacitated vehicles and capacitated depots, where each vehicle can be associated with a single depot to satisfy the requests of customers. This version with a set of benchmark data has been introduced by Sambola et al. [1]. They solve the problem with the tabu search (TS) method and give a lower bound by solving knapsack problem and asymmetric traveling salesman problem ad hoc.

The LRP stated in this work can be defined as follows. Consider $I=\{1,\ldots,n\}$ the set of customers and $J=\{d_1,\ldots,d_m\}$ the set of potential depots. Each depot $j\in J$ is characterized by a limited capacity b_j and a fixed cost f_j of establishment. Each customer $i\in I$ has a non-negative demand q_i which is known in advance and should be satisfied. Moreover, each depot is associated with a single uncapacitated vehicle. Let c_{ij} $(i,j\in I\cup J)$ be the traveling cost between i and j. The LRP consists of opening a subset of depots and elaborates vehicle tours to visit the set of customer in order to minimize the total cost of location and delivery. Various real LRP applications exist including mail delivery [17], waste collection [19] and many others summarized in a survey paper [25].

Exact methods. The LRP is a NP-hard problem as it deals with two NP-hard subproblems, namely facility location problem (FLP) and vehicle routing problem (VRP). The first exact methods were suggested in [20,21]. In [20], a branch and bound algorithm is developed for the LRP with only one single facility to be established and without tour length restrictions. In [21], a branch and cut algorithm for the LRP with a fixed number of vehicles per depot is proposed.

Heuristics. In addition to exact methods, heuristic algorithms are elaborated to find its near-optimal solutions. An extensive literature on heuristics and metaheuristic techniques are developed to solve either LRP for capacitated depots, capacitated vehicles or for both. Numerous studies on LRP with both capacities on depots and vehicles, called capacitated location routing problems (CLRPs), were published. Duhamel et al. [8] proposed a hybridization of a greedy randomized adaptive search procedure (GRASP) and an evolutionary local search (ELS). A simulated annealing (SA) based heuristic is proposed in [33] where competitive results especially in terms of solution quality are provided. More recently, Derbel et al. [6] suggested a VNS for solving CLRP with capacitated depots and a capacitated homogenous vehicle fleet. Problem may be solved by clustering customers on the network first [5]. Albreda-Sambola et al. [1] propose a compact formulation for the LRP solving it by applying TS, while more recently, Derbel et al. [7] developed an iterated local search (ILS) to solve the problem.

Variable neighborhood search. In solving combinatorial and global optimization problems, many efforts are made in order to deal with the weakness of local search strategies that fall into a local optimum which can be of poor quality, without having the ability of leaving it. Many strategies are proposed to avoid such situation. Among the pioneer algorithms in 1983 were SA [18] and TS [11]. The main feature of those approaches was to accept nonimproving moves within the local search. Mladenović and Hansen [22] suggested the systematic change of the neighborhood within the local search to evade local optima traps. This leads to design of a metaheuristic

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called variable neighborhood search (VNS), which has been further developed in its various extensions. For a recent survey on VNS, see [15]. Here we briefly list some successful VNS applications, related to location and routing problems separately. VNS was firstly tested for the traveling salesman problem with and without backhauls, where GENIUS [22] was used as its local search. There are other applications in the literature including the dial-a-ride problem [26], the multi-depot vehicle routing problem with time windows [28], and many others. Several variants of location problems are solved efficiently with the VNS method. Hansen et al. [12] propose a VNS for solving the uncapacitated p-median problem. Based on this work, Fathali et al. [9] present a new version of VNS to solve the p-median problem with positive and negative weights. Recently, Fleszar et al. [10] have proposed an effective VNS for the capacitated p-median problem and Ilić et al. [16] solve the uncapacitated single allocation p-hub median problem using a general VNS. On the other hand, VNS is also applied on different contexts of routing problems, e.g., the vehicle routing problem with time windows [3], the capacitated arc routing with intermediate facilities [27]. For the recent survey on successful applications of VNS, see e.g., [15]. Thus, VNS is shown to be relevant for solving both location and routing problems.

Motivation. We here discuss our basic motivation to apply VNS on this problem. Let us recall the main similarities and differences between VNS and the two heuristics already used for solving LRP: ILS and TS. The ILS method, also known as fixed neighborhood search [4] can be seen as an extension of a multistart local search. It performs a perturbation of a local optima and takes the point obtained as the initial one for the local search (rather than the random selection, as in multistart strategy). The basic components of ILS are similar to those used within VNS. Those common ingredients are local search, shaking and acceptance criteria. However, our main motivation to apply VNS in solving LRP is based on the fact that performing shaking and local search using several neighborhoods must be more beneficial than using only one neighborhood structure [7]. The probability of finding a global optimum is higher since the global optimum is a local optimum for all particular neighborhood structures used. Compared with ILS and VNS, TS explores the solution space by using the memory to save the history of the search. TS explores a neighborhood extensively and introduces different mechanisms to store the visited solutions and avoid cycles. Sometimes the memory use could be time consuming, may involve many parameters and may be dependent on the problem instances [1].

Contribution. In this work, to the best of our knowledge, for the first time VNS based heuristic is suggested for solving LRP; we consider location and routing problems simultaneously. The main idea was to compromise between routing configuration in the local search and the location of depots in the perturbation or shaking mechanism. Within variable neighborhood descent (VND), we use five different local search heuristics, two of them suggested here for the first time. Several shaking (perturbation) algorithms are tested as well. Based on computational results, our approach outperforms recent metaheuristic approaches.

Outline. The rest of the paper is organized as follows. Section 2 introduces VNS background followed by the VND algorithm and presents the proposed VNS scheme. The computational study is reported in Section 3, where our proposed VNS variants and those of the TS [1] and the ILS [7] are compared. Finally, we conclude the paper with Section 4.

2. Variable neighborhood search for the LRP

VNS is shown to be a promising metaheuristic for solving several difficult problems. Consider a combinatorial problem to minimize a function f defined on a solution space X. To each solution $x \in X$, is associated a subset $\mathcal{N}(x) \subseteq X$ called the neighborhood of x. Let $\mathcal{N}_k$, $k \in \{1, \ldots, k_{max}\}$ be the set of the neighborhood structures selected to be explored during the search. For each solution x, $\mathcal{N}_k(x)$ will be the set of solutions of the kth neighborhood of x. Three basic facts ensure the success of running several neighborhoods within VNS as stated in [14,22]: (i) a local minimum with respect to one neighborhood is not necessarily so with another, (ii) a global minimum is a local minimum with respect to all possible neighborhood structures, (iii) for many problems local minima with respect to one or several neighborhoods are relatively close to each other.

Different versions of VNS are proposed in the literature depending on whether the use of these principles are deterministic or stochastic. Indeed, VNS consists of a randomized part as the selection of a neighbor is random (in the shaking phase), and a deterministic part where the local search is applied, immediately after shaking.

In general, VNS starts with an initial solution x and a set of neighborhoods $\mathcal{N}_k$, $k = 1, \ldots, k_{max}$. At each iteration, a random solution x' is computed with respect to the kth neighborhood, $\mathcal{N}_k(x)$. Then, a local search is applied to the solution x' to yield a second solution x''. If x'' is better than x, the solution is updated and the process continues with the first neighborhood $\mathcal{N}_1(x)$, otherwise the same steps are repeated with the next neighborhood, $\mathcal{N}_{k+1}$. The final solution found will be a local optimum with respect to all neighborhood structures. The VNS combines three basic steps: a stochastic phase, which is the shaking step that finds a random neighbor of the incumbent solution, a deterministic phase which represents the application of any local search algorithm, and the evaluation step which accepts only solutions that improve the objective (sometimes the fitness) function. Other variants of VNS are derived if some components are slightly transformed as shown in [13]. In fact, the VNS can be seen as a variable depth search if we make a move to the best neighborhood among the predefined ones. The different steps of VNS are presented in Algorithm 1.

Algorithm 1. Basic VNS.

Input: The set of neighborhood structures $\mathcal{N}_k$, for $k = 1, \ldots, k_{max}$
Initialization: Find an initial solution x;
repeat
  $k \leftarrow 1$;
  while $k \le k_{max}$ **do**
    $x' \leftarrow$ a **random neighbor** in $\mathcal{N}_k(x)$ /*Shaking*/
    $x'' \leftarrow$ Local Search(x') /*Local Search*/
    if $f(x'') < f(x)$ **then** /*Move or Not*/
      $x \leftarrow x''$; $k \leftarrow 1$;
    else
      $k \leftarrow k+1$ /*Neighborhood Change*/;
until *termination condition is met*
return x;

In this work, we first develop a VND which performs a deterministic change of the neighborhoods. Then we apply the General VNS that uses VND as a local search routine. We give further details in the next sections.

2.1. Solution representation

In comparison with traditional vehicle routing and location problems, the combined LRP is obviously more complex. Both location and routing decisions are closely related and need to be simultaneously solved. For this reason, a LRP solution contains both location and routing attributes. Indeed, a solution encoding is very important to make an effective algorithm. In our implementation, a feasible solution x is represented as a set of sequences π_j for