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Solution approaches for the cutting stock problem with setup cost

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ABSTRACT

In this paper, we study the *Cutting Stock Problem with Setup Cost* (CSP-S) which is a more general case of the well-known *Cutting Stock Problem* (CSP). In the classical CSP, one wants to minimize the number of stock items used while satisfying the demand for smaller-sized items. However, the number of patterns/setups to be performed on the cutting machine is ignored. In most cases, one has to find the trade-off between the material usage and the number of setups in order to come up with better production plans. In CSP-S, we have different cost factors for the material and the number of setups, and the objective is to minimize total production cost including both material and setup costs. We develop a mixed integer linear program and analyze a special case of the problem. Motivated by this special case, we propose two local search algorithms and a column generation based heuristic algorithm. We demonstrate the effectiveness of the proposed algorithms on the instances from the literature.

1. Introduction

The classical *Cutting Stock Problem* (CSP) addresses the problem of cutting stock materials of length W in order to satisfy the demand of smaller pieces with demand d_i and length w_i while minimizing the trim loss. CSP is first introduced by Kantorovich [22] and studied in one or more dimensions to deal with real-world applications in various industries such as the glass, fiber, paper and steel industries [11]. The main objective in CSP is to minimize total waste/material cost, and the number of setups (different cutting patterns) are usually ignored. However, in real world applications, the number of cutting patterns in a production plan has to be considered since each cutting pattern requires an additional setup on the cutting machine. This is especially important when doing the setup on the cutting machine is time-consuming and/or costly. For example, Chien and Deng [7] mention the importance of both material and setup cost in semiconductor industry, and several authors focus on minimizing the setup and waste cost simultaneously [5,8,10,13,20,33].

In this paper, we study a general version of the one-dimensional CSP, in which we try to minimize total production cost (setup + material cost) while cutting stock items of length W into demand items of smaller size (w_i) in order to satisfy the demand (d_i) for each item. Different from the classical CSP, in this problem the total production cost includes both material cost and setup cost. Every time a new cutting pattern is used, a setup cost is incurred to set up the cutting machine. We call this problem the *Cutting Stock Problem with Setup Cost* (CSP-S). CSP-S unifies a wide domain of packing/cutting stock problems. Note that CSP-S reduces to CSP when the setup cost is assumed to be zero. It reduces to the well-known *Bin Packing Problem* (BPP) [23,25,29] when material cost is negligible and demand of each item is 1. Finally, CSP-S reduces to *Pattern Minimization Problem* (PMP) [27,32] which focuses on minimizing the number of different cutting patterns with a given number of stock items to be cut to satisfy the demand when the material cost dominates the setup cost. According to the typology by Dyckhoff [12], CSP-S belongs to type 1/V/I/R, which means that it is a one-dimensional problem with an unlimited supply of stock materials of the same size and a set of demand items. According to the typology by Waescher et al. [35], CSP-S is an input (value) minimization assignment problem which has a weakly heterogeneous assortment.

The remainder of the paper is organized as follows. We discuss the related work in the literature in Section 2. In Section 3, we provide a formal definition of the problem, and present an example to illustrate the problem. A mixed integer non-linear assignment formulation and a mixed integer linear model are presented in Section 4. In Section 5, we discuss the complexity of the problem and analyze a special case of the problem. We propose two local search algorithms and a column generation based heuristic algorithm for CSP-S in Section 6. We conduct computational experiments to compare the performance of the proposed algorithms and the other methods in the literature. The results are presented in Section 7. Conclusion is provided in Section 8.

2. Literature review

The most related problems in the literature are *Cutting Stock Problem* (CSP) and *Pattern Minimization Problem* (PMP). Several

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heuristic and exact algorithms are developed to solve CSP and PMP. Gilmore and Gomory [16,17] formulate CSP as an integer program where the decision variables are the number of times each cutting pattern is used. In CSP, when the number of demand items increases, the possible number of patterns and decision variables increase exponentially. Therefore, Gilmore and Gomory propose a method where they first create the useful cutting patterns by solving an auxiliary problem, then they solve the linear programming (LP) relaxation and apply rounding procedure to a typically non-integer solution. This heuristic approach often results in an optimal integer solution. This algorithm is also known as *Column Generation* algorithm which is comprehensively explained in Ford and Fulkerson [15]. Valério de Carvalho [28] develops a combination of column generation algorithm and a branch-and-bound procedure to solve CSP. He formulates the problem as an arc flow model with some side constraints which also provides a better lower bound. Similar to the column generation algorithm developed by Gilmore and Gomory [17], the new model can be split into a master and a subproblem. At each node of the branch-and-bound tree, he first introduces branching constraints in the master problem and then generates columns. Instead of generating only one attractive arc (pattern), arc flow model introduces a set of arcs corresponding to valid cutting patterns which accelerates the column generation procedure.

Belov and Scheithauer [3] propose an exact cutting plane algorithm in combination with column generation to address CSP when stock items have different sizes. In general, a knapsack problem is solved to generate a new column [17]. In the proposed algorithm, a modified heuristic method is used to find new columns which is generally more difficult than solving a knapsack problem. Belov [4] discusses the known models of CSP and proposes several solution approaches like branch-and-price method and delayed pattern generation heuristic algorithm. Waescher and Gau [34] first discuss existing heuristic and exact methods to solve CSP by generating integer patterns and determining the production frequencies. Then, they propose several heuristic approaches to generate integer patterns for a relaxation of an integer model known as *Complete-Cut Model* [16,17,26]. The heuristics are tested on randomly generated instances in comparison with optimal solutions which are found using the proposed lower and upper bounds on the optimal solution.

In general, PMP can be defined as minimizing the number of different cutting patterns while satisfying the demand with a given number of stock items [32]. However, most of the authors use CSP solution to determine the number of stock items, say K, and look for a solution with minimum number of different patterns using K stock items. McDiarmid [24] proves that PMP is strongly NP-hard even for a special case where only two items fit into a stock item but none of the three do, and proposes a heuristic approach to find packings of balanced subsets in order to solve this special case of PMP. Alves and Valério de Carvalho [2] develop a branch-and-price-and-cut algorithm after formulating PMP using an arc flow idea. They use dual feasible functions and linear combinations of added constraints to derive new valid inequalities and strengthen the bounds of the column generation algorithm. Vanderbeck [32] develops a mixed integer formulation for PMP and solves it using a column generation approach with linear master problem and non-linear subproblem which is decomposed into bounded knapsack problems. In addition, a branch-and-bound procedure with several super-additive inequalities to tighten the feasible region is proposed. Belov [4] modifies Vanderbeck's compact mixed integer formulation to strenghten the LP relaxation bound and then proposes a simple non-linear model for PMP. His method is based on using an enumerative scheme for LP relaxation model. Umetani et al. [27] propose a heuristic algorithm, called *Iterated Local Search with Adaptive Pattern Generation* (ILS-APG), to minimize the number of different cutting patterns used in CSP. They first assume that the number of patterns is fixed, say K patterns, and then search for a solution with a minimum quadratic objective which shows the deviation of production from demand. Afterwards, they try to minimize K iteratively using ILS-APG.

Foerster and Waescher [14] propose a heuristic method to minimize the number of different patterns for minimum waste solution. This heuristic approach has two steps: (i) finding a minimum waste solution regardless of setup cost, and (ii) combining any p patterns and substitute them with $p-1$ patterns for $p \leq 4$ using a method called KOMBI. Cui et al. [8] present a sequential heuristic algorithm, called *Sequential Heuristic Procedure - Cui* (SHPC), to solve PMP. This algorithm generates a cutting pattern using a subset of unassigned items, decides the production frequency and repeats until all items are assigned. Cui and Liu [9] modify SHPC by introducing two candidate sets for pattern selection. The first set of candidate items can be used for finding a new pattern, and the second set includes all the previously generated patterns. The final set of patterns are chosen considering the total material and setup cost. Yanasse and Limeira [36] propose a hybrid scheme to heuristically reduce the number of different patterns in CSP with any dimension. They first generate patterns and select good patterns using some predefined criteria and update the set of items whose demands are not satisfied. Then, they solve the reduced problem using the LP relaxation. Fractional values are rounded down to nearest integer, and the remaining demand is satisfied using *First Fit Decreasing* packing algorithm [21]. Finally, they apply the KOMBI heuristic proposed by Foerster and Waescher [14] to combine several patterns.

Finally, Farley and Richardson [13], Walker [33], Haessler [20], Belov and Scheithauer [5] also study problems related to CSP-S where both material and setup cost are considered. Farley and Richardson [13] introduce this problem as a subset of the general fixed-charge problem to solve the two-dimensional trim-loss problem in the glass industry. In Farley and Richardson [13], similar to Walker [33], an initial solution for a linear model of CSP with fixed charge per setup is produced. Then, a new pattern is allowed into basis (using simplex method), only if the total production cost is decreased. Afterwards, a heuristic algorithm is used to improve the solution by swapping cutting patterns. They use the algorithm by Gilmore and Gomory [18] to generate the patterns throughout the algorithm. Haessler [20] introduces a mixed integer pattern-based formulation to solve the one-dimensional fixed-charge problem with lower and upper bounds on the customer order requirements. He proposes a sequential heuristic approach and uses manually solved small instances to evaluate the performance of the proposed approach. Belov and Scheithauer [5] design a sequential heuristic approach to minimize the number of input stock materials and present the effectiveness of this algorithm to minimize the number of different cutting patterns and the maximum number of open stacks during the cutting process.

3. Problem definition

In this section, we first provide a formal definition of the problem, and then, give an example to illustrate the differences between CSP, PMP, and CSP-S. In CSP-S, we have stock items of length W and a set of demand items with given length (w_i) and demand (d_i) values. We use $I = \{1, \ldots, n\}$ to denote the set of demand items. We satisfy the demand for each item by cutting stock items according to some cutting patterns. For each cutting pattern, we incur a setup cost of C_s. Finally, C_p denotes the cost of a stock item per unit length. Our objective is to satisfy the