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Heuristics for the multi-item capacitated lot-sizing problem with lost sales

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ABSTRACT

This paper deals with the multi-item capacitated lot-sizing problem with setup times and lost sales. Because of lost sales, demands can be partially or totally lost. To find a good lower bound, we use a Lagrangian relaxation of the capacity constraints, when single-item uncapacitated lot-sizing problems with lost sales have to be solved. Each subproblem is solved using an adaptation of the $O(T^2)$ dynamic programming algorithm of Aksen et al. [5]. To find feasible solutions, we propose a non-myopic heuristic based on a probing strategy and a refining procedure. We also propose a metaheuristic based on the adaptive large neighborhood search principle to improve solutions. Some computational experiments showing the effectiveness and limitation of each approach are presented.

1. Introduction

Production planning consists in deciding how to transform raw materials or semi-finished products into final products in order to satisfy demands on time and at minimal cost. Determining lot sizes is a crucial decision in production planning; which consists in calculating the quantity to produce for each item at each time period. In industrial contexts, several constraints may complicate the problem. In particular, the fact that items need a resource makes the problem more complex. Indeed, this can lead to the impossibility to entirely satisfy demands when there is not enough capacity. Such an amount of unsatisfied demands is referred to as *shortage on demand* or *lost sales*.

In this paper, we address the single-level, single-resource, Multi-item Capacitated Lot-Sizing problem with setup times and Lost Sales called MCLS-LS. MCLS-LS consists in planning the production of N items over a horizon of T periods in order to satisfy time-varying demands. Demands are given for each item at each period. The different parameters of the problem are as follows. Producing a unit of item i in period t incurs a production cost α_{it} and requires ν_{it} units of the total capacity C_t. The holding cost of a unit of product i at the end of period t is γ_{it}. Product dependent setup costs β_{it} and setup times f_{it} are incurred at each period where production takes place. The problem has the distinctive feature of allowing demand shortages (also called lost sales). Lost sales are particularly relevant in problems with tight capacities. Indeed, when the available resources are not sufficient to produce the total demand, the capacity is spread among the items by minimizing the total lost sales. Thus, we introduce in the model a unitary lost sales cost φ_{it} for item i at period t. These costs should be viewed as penalty costs and their values are high compared to other unitary cost components. The objective is to minimize total production, setup, holding, and lost sale costs.

The classical Multi-item Capacitated Lot-Sizing problem with setup times (MCLS) is strongly NP-Hard [10]. Even the single item capacitated lot-sizing problem is NP-Hard in the ordinary sense [8]. The MCLS-LS problem is then NP-Hard. In fact, if we set lost sale costs to sufficiently high values, the problem to solve becomes the classical MCLS problem. Lot-sizing problems have been studied for five decades, with numerous references dealing with capacitated lot-sizing problems. For a recent survey on lot-sizing problems, the reader can refer to [17,18] for Multi-item Capacitated Lot-Sizing problems, and [9] for single-item capacitated lot-sizing problems.

Different approaches were addressed in the literature in order to find near optimal heuristic solutions for the MCLS problem. Trigeiro et al. [26] were among the first to solve the MCLS problem. They propose a smoothing heuristic based on Lagrangian relaxation of the capacity constraints. At each step of the subgradient method, a heuristic is called to obtain a feasible solution from the current Lagrangian lower bound. Belvaux and Wolsey [6], Pochet and Wolsey [24] and Miller et al. [20] propose exact methods to solve MCLS problems by strengthening the Linear Programming (LP) relaxation using valid inequalities. Recently, Degraeve and Jans [12] propose a new Dantzig–Wolfe reformulation and branch-and-price algorithm for the MCLS problem.

There has been little research on lot-sizing problems with demand shortages or lost sales. Sandbothe and Thompson [25] introduced the concept of shortages to the classical uncapacitated

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single-item lot-sizing problem, they called it stockouts. The authors propose an $O(T^3)$ dynamic programming algorithm to solve the problem optimally. Aksen et al. [5] address the same problem but called this concept *lost sales*. They improved the previous complexity by proposing an $O(T^2)$ dynamic programming algorithm. Hwang and van den Heuvel [14] propose an $O(T^4)$ dynamic programming algorithm to solve optimally the classical uncapacitated single-item lot-sizing problem with lost sales, upper bounds on stocks and concave costs. They also propose an algorithm in $O(T \log T)$ and $O(T)$ to solve respectively the uncapacitated lot-sizing problem with lost sales and non-speculative cost structure, and the same problem with nonincreasing selling prices. Absi et al. [4] address the uncapacitated single-item lot-sizing with time windows, early productions, lost sales and backlogging. The authors develop $O(T^2)$ solving algorithms for different variants of this problem.

Berk et al. [7] study the single-item lot-sizing problem for a warm/cold process with immediate lost sales by defining some properties of the optimal solutions. Recently, Absi and Kedad-Sidhoum [2] propose a branch-and-cut algorithm to solve the MCLS-LS problem with production time windows. They use a generalized version of Miller et al. [20] valid inequalities to strengthen the LP relaxations in the branch-and-bound tree. Absi and Kedad-Sidhoum [1,3] develop respectively MIP-Based heuristics to solve the MCLS-LS problem with additional industrial constraints and a Lagrangian relaxation approach to solve the MCLS-LS problem with safety stocks.

The main contributions of this paper are twofold. First, we adapt the dynamic programming algorithm of Aksen et al. [5] to take any cost structure into account and use this algorithm in a Lagrangian relaxation approach to find good lower bounds. Second, we develop new Lagrangian heuristics based on a smoothing algorithm and a probing strategy to find near-optimal solutions. Generally, the smoothing heuristics that are presented in the literature are myopic. We show through our computational experiments (Section 5.1) that these heuristics are no longer competitive with recent versions of commercial mathematical programming solvers. The algorithm proposed in this paper is non-myopic, i.e. a probing heuristic is used at each step to evaluate promising moves. It provides better results than classical smoothing heuristics. We also show the efficiency of the adaptive local search principle in improving lot-sizing solutions. This principle already shows its performance in solving several vehicle routing problems [22,23].

Section 2 describes MIP formulations of the MCLS-LS problem. In Section 3, we present a Lagrangian relaxation approach based on the relaxation of capacity constraints, an adaptation of the dynamic program proposed by Aksen et al. [5] to solve the single item uncapacitated version of MCLS-LS, and a subgradient method. Section 4 describes the principle of the Lagrangian heuristics based on a probing strategy as well as a refining procedure. We propose 14 neighbor operators and integrate them in a metaheuristic (Adaptive Local Search) based on the selection principle of the adaptive large neighborhood search [22] to improve solutions. Computational experiments are shown in Section 5 to evaluate the effectiveness and limit of our procedures. Finally, Section 6 provides a short conclusion and future research directions.

2. Mathematical formulation of the MCLS-LS problem

Different mathematical formulations were presented in the literature for the MCLS problem. Two mathematical formulations of the MCLS-LS problem are recalled in this section. The first one is a generalization of the classical formulation of the MCLS problem usually called *aggregate formulation*. The second formulation is based on the facility location formulation initially proposed by Krarup and Bilde [19] for the uncapacitated single-item problem, which is often called *disaggregate formulation*.

2.1. An aggregate formulation

In the following, we present an aggregate formulation of the MCLS-LS problem. This formulation is addressed in several papers such as Trigeiro et al. [26]. The notations are given below:

Sets and indices

T: Number of periods.
$\mathcal{T} = \{1, \ldots, T\}$.
N: Number of items.
$\mathcal{I} = \{1, \ldots, N\}$.
i: index of an item, $i = 1, \ldots, N$.
t: index of a period, $t = 1, \ldots, T$.

Parameters:

d_{it}: demand for item i at period t.
C_t: available capacity at period t.
f_{it}: setup time for item i at period t.
ν_{it}: unitary resource consumption for item i at period t.
α_{it}: production unit cost for item i at period t.
β_{it}: setup cost for item i at period t.
γ_{it}: inventory unit cost for item i at period t.
φ_{it}: lost sale unit cost for item i at period t.

Variables:

x_{it}: the quantity of item i produced in period t.
y_{it}: binary setup variable, equal to 1 if item i is produced at period t (i.e. $x_{it} > 0$), and 0 otherwise.
s_{it}: the inventory level of item i at the end of period t.
r_{it}: the lost sales of item i at period t.

The aggregate formulation (noted AGG) of the MCLS-LS model is stated as follows:

$$\min \sum_{i \in \mathcal{I}, t \in \mathcal{T}} (\alpha_{it} x_{it} + \beta_{it} y_{it} + \varphi_{it} r_{it} + \gamma_{it} s_{it}) \quad (1)$$

$$s_{i,t-1} + r_{it} + x_{it} = d_{it} + s_{it} \quad \forall i,t \quad (2)$$

$$\sum_{i \in \mathcal{I}} (\nu_{it} x_{it} + f_{it} y_{it}) \le c_t \quad \forall t \quad (3)$$

$$x_{it} \le M_{it} y_{it} \quad \forall i,t \quad (4)$$

$$r_{it} \le d_{it} \quad \forall i,t \quad (5)$$

$$x_{it}, r_{it}, s_{it} \ge 0 \quad \forall i,t \quad (6)$$

$$y_{it} \in \{0,1\} \quad \forall i,t \quad (7)$$

The objective function (1) minimizes the total cost that aggregates production, setup, inventory and shortage costs. Constraints (2) are the inventory balance equations. Constraints (3) are the capacity constraints; the overall consumption must be lower than or equal to the available capacity. Constraints (4) relate the binary setup variables to the continuous production variables. M_{it} can be set to $\min\{\sum_{t'=t}^{T} d_{it'}, (c_t - f_{it})/\nu_{it}\}$. Constraints (5) define upper bounds on the lost sale variables. The domain definitions of the variables are defined in Constraints (6) and (7).