Computers & Operations Research 40 (2013) 362–377

Contents lists available at SciVerse ScienceDirect

Computers & Operations Research

journal homepage: www.elsevier.com/locate/caor

Scenario Cluster Decomposition of the Lagrangian dual in two-stage stochastic mixed 0–1 optimization

Laureano F. Escudero [a], M. Araceli Garín [b,*], Gloria Pérez [c], Aitziber Unzueta [b]

[a] Dpto. Estadística e Investigación Operativa, Universidad Rey Juan Carlos, Spain
[b] Dpto. de Economía Aplicada III, Universidad del País Vasco, Spain
[c] Dpto. de Matemática Aplicada y Estadística e I.O., Universidad del País Vasco, Spain

ARTICLE INFO

Available online 14 July 2012

Keywords:
Two-stage stochastic integer programming
Nonanticipativity constraints
Cluster Lagrangian Decomposition
Scenario cluster model
Subgradient Method
Volume Algorithm
Progressive Hedging Algorithm
Dynamic Constrained Cutting Plane Scheme

ABSTRACT

In this paper we introduce four scenario Cluster based Lagrangian Decomposition procedures for obtaining strong lower bounds to the (optimal) solution value of two-stage stochastic mixed 0–1 problems. At each iteration of the Lagrangian based procedures, the traditional aim consists of obtaining the solution value of the corresponding Lagrangian dual via solving scenario submodels once the nonanticipativity constraints have been dualized. Instead of considering a splitting variable representation over the set of scenarios, we propose to decompose the model into a set of scenario clusters. We compare the computational performance of the four Lagrange multiplier updating procedures, namely the Subgradient Method, the Volume Algorithm, the Progressive Hedging Algorithm and the Dynamic Constrained Cutting Plane scheme for different numbers of scenario clusters and different dimensions of the original problem. Our computational experience shows that the Cluster based Lagrangian Decomposition bound and its computational effort depend on the number of scenario clusters to consider. In any case, our results show that the Cluster based Lagrangian Decomposition procedures outperform the traditional Lagrangian Decomposition scheme for single scenarios both in the quality of the bounds and computational effort. All the procedures have been implemented in a C++ experimental code. A broad computational experience is reported on a test of randomly generated instances by using the MIP solvers COIN-OR (2010, [18]) and CPLEX (2009, [17]) for the auxiliary mixed 0–1 cluster submodels, this last solver within the open source engine COIN-OR. We also give computational evidence of the model tightening effect that the preprocessing techniques, cut generation and appending and parallel computing tools have in stochastic integer optimization. Finally, we have observed that the plain use of both solvers does not provide the optimal solution of the instances included in the testbed with which we have experimented but for two toy instances in affordable elapsed time. On the other hand the proposed procedures provide strong lower bounds (or the same solution value) in a considerably shorter elapsed time for the quasi-optimal solution obtained by other means for the original stochastic problem.

1. Introduction

In this work we consider a general two-stage stochastic mixed 0–1 problem. The uncertainty is modeled via a finite set of scenarios $\omega = 1, \ldots, |\Omega|$, each with an associated probability of occurrence w^{ω}, $\omega \in \Omega$. The traditional aim in this type of problems is to solve the so-called Deterministic Equivalent Model (DEM), which is a mixed 0–1 problem with a special structure, see e.g., [22] for a good survey of some major results in this area obtained during the 90s and beyond. A Branch-and-Bound algorithm for solving problems having mixed-integer variables in both stages is designed in [5], among others, by using Lagrangian relaxation for obtaining lower bounds to the optimal solution of the original problem. A Branch-and-Fix Coordination (BFC) methodology for solving such DEM in production planning under uncertainty is given in [1,2], but the approach does not allow continuous first stage variables or 0–1 second stage variables. We propose in [6,7] a BFC algorithmic framework for obtaining the optimal solution of the two-stage stochastic mixed 0–1 integer problem, where the uncertainty appears anywhere in the coefficients of the 0–1 and continuous variables in both stages. Recently, a general algorithm for two-stage problems has been presented in [23].

We study in [11] several solution methods for solving the dual problem corresponding to the Lagrangian Decomposition (LD) of two-stage stochastic mixed 0–1 models. At each iteration of these

* Corresponding author.
E-mail addresses: laureano.escudero@urjc.es (L.F. Escudero), mariaaraceli.garin@ehu.es (M. Araceli Garín), gloria.perez@ehu.es (G. Pérez), aitziber.unzueta@ehu.es (A. Unzueta).

0305-0548/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cor.2012.07.009

Lagrangian based procedures, the traditional aim consists of obtaining the solution value of the corresponding parametric mixed 0–1 Lagrangian dual problem via solving single scenario submodels once the nonanticipativity constraints (NAC) have been dualized, and the parameters (i.e., the Lagrange multipliers) are updated by using different subgradient and cutting plane based methodologies.

Instead of considering a splitting variable representation over the set of scenarios, in this paper we propose a new approach so named Cluster Lagrangian Decomposition (for short, CLD) to decompose the model into a set of scenario clusters. So, we computationally compare the performance of the Subgradient Method (SM) [16], the Volume Algorithm (VA) [3], the Progressive Hedging Algorithm (PHA) [21] and the Dynamic Constrained Cutting Plane (DCCP) scheme [19] for Lagrange multipliers updating while solving large-scale stochastic mixed 0–1 problems in an algorithmic framework based on scenario clusters decomposition. A successful result may open up the possibility for tightening the lower bounds of the solution value at the candidate Twin Node Families in the exact BFC scheme for both two-stage and multistage types of problems, see e.g., [8].

For different choices of the number of scenario clusters we report the computational experience by using CPLEX, integrated in the COIN-OR environment, to verify the effectiveness of the proposal. In this sense, we also give computational evidence of the model tightening effect and the computational cost that preprocessing, cut generation and appending and parallel computing tools have in stochastic integer optimization too, see [20]. We also computationally compare the new approach with the cluster singleton one (i.e., the LD for single scenarios). It outperforms it as well as the plain use of the MIP solver of choice, CPLEX. The proposed approach provides a tight lower bound such that the quasi-optimality gap of the upper solution bound obtained by other means on large-scale instances is very small and frequently, guarantees its optimality. However, in some cases the plain use of CPLEX does not even provide the feasible solution within a large elapsed time limit, such that its objective function value is simply an upper bound of the solution value of the original stochastic problem. Additionally, in other cases where the plain use of CPLEX obtains feasible solutions without guaranteeing their optimality within the time limit, we can prove in much smaller elapsed time that the incumbent CPLEX solution is the optimal one, since our CLD procedures provide lower bounds identical to the value of that solution. Finally, the CPLEX incumbent solution is also frequently worse than the solution that is obtained by our CLD approach, being the quality of the solution and the small elapsed time that it requires good enough.

The remainder of the paper is organized as follows: Section 2 presents the two-stage stochastic mixed 0–1 problem in compact and splitting variable representations over the scenarios and scenario clusters. Section 3 summarizes the theoretical results on Lagrangian decomposition and presents the Cluster Lagrangian Decomposition approach. Section 4 presents the four procedures mentioned above for updating the Lagrange multipliers. Section 5 reports the results of the computational experiment. Section 6 concludes.

2. Two-stage stochastic mixed 0–1 problem

In many real cases a two-stage deterministic mixed 0–1 optimization model must be extended to consider the uncertainty in some of the parameters. In our case, these are the objective function, the right and left hand-side vectors and the constraint matrix coefficients. This uncertainty is introduced by using the scenario analysis approach. When a finite number of scenarios is considered, a general two-stage program can be expressed in terms of the first stage decision variables being equivalent to a large, dual block-angular programming problem, introduced in [26] and known as Deterministic Equivalent Model (DEM). It is worth to point out that the uncertainty of the second-stage parameters affects not only the second-stage variables but also the first-stage ones.

Let us consider the *compact* representation of the DEM of a two-stage stochastic integer problem (*MIP*),

$$
\begin{aligned}
(MIP)^c: z_{MIP} = \min\ & c_1\delta + c_2 x + \sum_{\omega\in\Omega} w^\omega [q_1^\omega \gamma^\omega + q_2^\omega y^\omega] \\
\text{s.t. } & b_1 \le A\begin{pmatrix}\delta\\ x\end{pmatrix} \le b_2 \\
& h_1^\omega \le T^\omega \begin{pmatrix}\delta\\ x\end{pmatrix} + W^\omega \begin{pmatrix}\gamma^\omega\\ y^\omega\end{pmatrix} \le h_2^\omega, \quad \forall \omega\in\Omega \\
& \delta,\gamma^\omega \in \{0,1\}, \quad x,y^\omega \ge 0,\ \forall\omega\in\Omega,
\end{aligned} \tag{1}
$$

where the uncertainty may affect parameters associated with all variables (first-stage and second-stage variables). c_1 and c_2 are known vectors of the objective function coefficients for the δ and x variables in the first stage, respectively, b_1 and b_2 are the known left and right hand side vectors for the first stage constraints, respectively, and A is the known matrix of coefficients for the first stage constraints. For each scenario ω, w^ω is the likelihood attributed to the scenario, such that $\sum_{\omega\in\Omega} w^\omega = 1$, h_1^ω and h_2^ω are the left and right hand side vectors for the second stage constraints, respectively, and q_1^ω and q_2^ω are the objective function coefficients for the second stage γ and y variables, respectively, while T^ω and W^ω are the technology and recourse constraint matrices under scenario ω, for $\omega\in\Omega$, where Ω is the set of scenarios to consider. Notice that there are two types of decision variables at each stage, namely, the set of δ 0–1 and x continuous variables for the first stage, and the set of γ^ω 0–1 and y^ω continuous variables for the second stage.

Notice also that for the purpose of simplification, the objective function to optimize in the models dealt with in this paper is the expected value over the set of scenarios Ω, i.e., the risk neutral attitude. An interesting extension appears, in case of considering other coherent risk averse measures as opposed to the risk neutral attitude considered in this work, like the VaR, CVaR or even the optimization of the objective function expected value subject to stochastic dominance constraints (sdc) for a set of profiles, see e.g., [9,10]. In all of these models appear the known as scenario linking constraints, which must be suitably treated in the scenario-cluster decomposition of the model.

The structure of the uncertain information can be visualized as a tree, where each root-to-leaf path represents one specific scenario, ω, and corresponds to one realization of the whole set of the uncertain parameters. In the example depicted in Fig. 1, there are $|\Omega| = 10$ root-to-leaf possible paths, i.e., scenarios. Following the nonanticipativity principle, stated in [26] and restated in [21], all scenarios should have the same value for the related first stage variables in the two-stage problem.

The left section of Fig. 1 implicitly represents the nonanticipativity constraints (NAC, for short). This is the compact representation shown in model (1). The right section of Fig. 1 gives the same information as the compact representation but using a splitting variable scheme and noticing that it explicitly represents the NAC (i.e., imposing the equality) on the first stage variables δ^ω, x^ω and for all the scenarios ω.

Let us consider the *splitting variable* representation of the DEM of the two-stage stochastic mixed 0–1 problem:

$$
(MIP)^s:\ z_{MIP} = \min \sum_{\omega\in\Omega} w^\omega [c_1\delta^\omega + c_2 x^\omega + q_1^\omega\gamma^\omega + q_2^\omega y^\omega]
$$