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Minimising maximum response time

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ABSTRACT

The minmax response time problem (mRTP) is a scheduling problem that has recently appeared in the literature and can be considered as a fair sequencing problem. This kind of problems appears in a wide range of real-world applications in mixed-model assembly lines, computer systems, periodic maintenance and others. The mRTP arises whenever products, clients or jobs need to be sequenced in such a way that the maximum time between the points at which they receive the necessary resources is minimised. The mRTP has been solved in the literature with a greedy heuristic. The objective of this paper is to improve the solution of this problem by means of exact and heuristic methods. We propose one mixed integer linear programming model, nine local search procedures and five metaheuristic algorithms. Extensive computational experiments are carried out to test them.

1. Introduction

The concept of *fair sequence* has emerged independently from scheduling problems in a variety of real-life context: among others, mixed-model assembly lines (e.g., [1]), computer multi-threaded systems (e.g., [2]), apportionment of the seats of a House among the states (e.g., [3]), periodic machine maintenance (e.g., [4]), commercial advertisements airings (e.g., [5]) and waste collection [6]. The common aim of these scheduling problems, as defined in [7], is to build a fair sequence using n symbols, where symbol i (i=1,…,n) must be copied d_i times in the sequence. The fair sequence is the one which allocates a fair share of positions to each symbol i in any subsequence. This *fair* or *ideal* share of positions allocated to symbol i in a subsequence of length h is proportional to the relative importance (d_i) of symbol i with respect to the total copies D of competing symbols ($D=\sum_{i=1..n} d_i$). There is no universal definition of fairness, as several reasonable metrics can be defined according to the specific problem considered. For a detailed introduction to fair sequences, see [8].

In particular, the metric known as *response time* refers to the time that events, jobs, clients, etc. wait for their next turn in obtaining the resources they need to advance [9].

Salhi and García-Villoria [10] have recently introduced the minmax response time problem (mRTP). It lies in finding the sequence that minimises the maximum response time (maxRT). Formally, let $\bar{t}_i$ be the ideal or average distance between two consecutive copies of symbol i ($\bar{t}_i = D/d_i$). And let S be a solution of an instance in the mRTP that consists of a circular sequence of D copies ($S=s_1s_2\ldots s_D$), where s_h is the copy placed in position h of sequence S. For each symbol i in which $d_i \geq 2$, let t_k^i be the distance between the positions in which the copies k+1 and k of symbol i are found. We consider the distance between two consecutive positions to be equal to 1. Since the sequence is circular, position 1 comes immediately after the last position D; therefore, $t_{d_i}^i$ is the distance between the first copy of symbol i in a cycle and the last copy of the same symbol in the preceding cycle. And for all symbol i in which $d_i=1$, t_1^i is equal to $\bar{t}_i$. The maxRT is the maximum absolute error with respect to the ideal distances; that is, $\text{maxRT} = \max_{i=1}^{n} \max_{k=1}^{d_i} |t_k^i - \bar{t}_i|$. Note that the symbols i in which $d_i=1$ do not intervene in the computation of maxRT.

For an illustration, consider that $n=3$ with symbols A, B and C. Also consider $d_A=3$, $d_B=2$ and $d_C=2$; thus, $D=7$, $\bar{t}_A=7/3$, $\bar{t}_B=7/2$ and $\bar{t}_C=7/2$. Any sequence that contains exactly d_i times each symbol $i \in \{A, B, C\}$ is a feasible solution. For example, the sequence (A, B, A, C, B, C, A) is a feasible solution, where

$$\text{maxRT} = \max(\max(|2-7/3|, |4-7/3|, |1-7/3|),$$
$$\max(|3-7/2|, |4-7/2|), \max(|2-7/2|, |5-7/2|)) = 5/3.$$

Real-life minmax problems are habitually treated in the scientific literature; for instance, location [11], assembly line balancing [12] and other production problems [13] have minmax objectives.

A problem related to the mRTP, known as response time variability problem (RTVP), has extensively treated in the last years. It consists in finding the sequence that minimises the response time variability (RTV): $\text{RTV} = \sum_{i=1}^{n} \sum_{k=1}^{d_i} (t_k^i - \bar{t}_i)^2$. The RTVP has been solved by means of mixed integer linear programming (MILP) [9,14], branch and bound (B&B) [15], heuristics [10,16], metaheuristics [17] and hyper-heuristics [18].

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On the other hand, to the best of our knowledge, there is only one reference of the mRTP. In [10], the mRTP is introduced and solved with a greedy heuristic, but any indicator of the quality of the obtained solutions is given. Although the heuristic proposed in [10] was originally designed for the resolution of the RTVP, the authors could use it to solve the mRTP since a feasible solution of one problem is also a feasible solution to the another problem (note that, in both problems, a solution consists of a sequence of d_i copies of each symbol i). However, a good solution for the RTVP is not necessarily a good solution for the mRTP (and vice versa).

Note that the mRTP can be seen as the minmax version of the RTVP. In some contexts, it is more suitable to solve a problem as a mRTP rather than RTVP; for instance, in a context of preventive maintenance. A worker has to grease n machines and each machine i needs to be greased d_i times during a horizon of $D=\sum_{i=1..n} d_i$ unit times. The average time between two consecutive visits to machine i is $\bar{t}_i = D/d_i$. In the cyclic schedule for the grease task, it is usually desired that the maximum difference, for all machines, between $\bar{t}_i$ and the interval time of two consecutive maintenance services to machine i is as small as possible in order to reduce the probability of any critical failure.

The objective of this paper is to design exact and heuristic methods specifically designed to solve the mRTP in order to overcome the procedures existing in the literature. Specifically, we propose one MILP model, nine local search procedures and five metaheuristic algorithms. Because of the link between the mRTP and the RTVP, we can take advantage of the RTVP literature to design our methods for the mRTP.

The remainder of this paper is organised as follows. Section 2 proposes a MILP model for the mRTP and a lower bound on the value of the objective function. Section 3 provides several local search procedures and metaheuristic algorithms for solving larger instances beyond the scope of the MILP model. An extensive computational experiment is carried out, whose results are presented in Section 4. Finally, Section 5 gives some conclusions with highlights for future research.

2. Mathematical formulation and a lower bound

The proposed MILP model uses a lower bound (LB) on the mRT value that is based on the fact that the real distances between copies (t_k^i) are integer and the ideal distances ($\bar{t}_i$) may be real. That lower bound (LB) is calculated as follows:

$$LB = \max\left(\max_{i=1}^{n}(\lceil \bar{t}_i \rceil - \bar{t}_i), \max_{i=1}^{n}(\bar{t}_i - \lfloor \bar{t}_i \rfloor)\right) \quad (1)$$

where $\lceil x \rceil$ is the operator that returns the smallest integer that is equal to or greater than x and $\lfloor x \rfloor$ is the operator that returns the greatest integer that is equal to or smaller than x.

A first non linear mathematical model is formulated as follows.

Data:

- n number of symbols.
- d_i number of copies of symbol i to be scheduled ($i=1,\dots,n$).
- D total number of copies, which is also the number of positions in the sequence: $D=\sum_{i=1}^{n} d_i$.
- $\bar{t}_i$ ideal distance between two consecutive copies of symbol i ($i=1,\dots,n$): $\bar{t}_i = D/d_i$.
- $G1$ set of symbols with multiple copies: $G1=\{i=1,\dots,n : d_i \geq 2\}$.
- $i*$ symbol with the lowest d_i value such as $d_i \geq 2$: $i* = \operatorname{argmin}_{i \in G1} d_i$.
- E_{ik}, L_{ik} the earliest and the latest position that can be occupied by copy k of symbol i ($i \in G1; k=1,\dots,d_i$): $E_{ik}=k, L_{ik}=D-d_i+k$.
- H_{ik} set of positions that can be occupied by copy k of symbol i ($i \in G1;\ k=1,\dots,d_i$): $H_{ik}=\{h=E_{ik},\dots,L_{ik}\}$.
- UB_i upper bound on the maximum response time of symbol i ($i \in G1$): $UB_i=(D-d_i+1)-\bar{t}_i$.
- V ordered set of possible values of the objective function: $V=V1 \cup V2$, where $V1=\{\lceil \bar{t}_i \rceil - \bar{t}_i + j : i \in G1, j=\lceil LB-(\lceil \bar{t}_i \rceil - \bar{t}_i)\rceil, \dots, \lfloor UB_i - (\lceil \bar{t}_i \rceil - \bar{t}_i)\rfloor\}$ and $V2=\{\bar{t}_i - \lfloor \bar{t}_i \rfloor + j : i \in G1, j=\lceil LB-(\bar{t}_i - \lfloor \bar{t}_i \rfloor)\rceil, \dots, \lfloor UB_i - (\bar{t}_i - \lfloor \bar{t}_i \rfloor)\rfloor\}$.
- v_f the f-th value in the set V ($f=1,\dots,|V|$).

Variables:

- mrt value of the objective function, $LB \leq mrt \leq \max_{i \in G1} UB_i$.
- $y_{ikh} \in \{0,1\}$ 1 if and only if copy k of symbol i is placed in position h ($i \in G1;\ k=1,\dots,d_i;\ h \in H_{ik}$).
- $w_f \in \{0,1\}$ 1 if and only if the objective function value is equal to v_f ($f=1,\dots,|V|$).

Model:

$$[MIN]\ Z = mrt \quad (2)$$

$$\sum_{i \in G1} \sum_{\substack{k=1 \mid \\ h \in H_{ik}}}^{d_i} y_{ikh} \leq 1 \qquad h=1,\dots,D \quad (3)$$

$$\sum_{h \in H_{ik}} y_{ikh} = 1 \quad i \in G1 \qquad k=1,\dots,d_i \quad (4)$$

$$y_{i*11} = 1 \quad (5)$$

$$1 + \sum_{h \in H_{ik}} h \cdot y_{ikh} \leq \sum_{h \in H_{i,k+1}} h \cdot y_{i,k+1,h} \quad i \in G1; k=1,\dots,d_i-1 \quad (6)$$

$$mrt = \max\left(\begin{array}{l} \max_{i \in G1} \max_{k=1}^{d_i-1}\left(\left|\left(\sum_{h \in H_{i,k+1}} h \cdot y_{i,k+1,h} - \sum_{h \in H_{ik}} h \cdot y_{ikh}\right) - \bar{t}_i\right|\right), \\ \max_{i \in G1}\left(\left|\left(D - \sum_{h \in H_{i,d_i}} h \cdot y_{i,d_i,h} + \sum_{h \in H_{i1}} h \cdot y_{i1h}\right) - \bar{t}_i\right|\right) \end{array}\right) \quad (7)$$

$$mrt = \sum_{f=1}^{|V|} v_f \cdot w_f \quad (8)$$

$$\sum_{f=1}^{|V|} w_f = 1 \quad (9)$$

Objective function (2) minimises the maximum response time. Constraints (3) and (4) ensure, respectively, that no more than one copy of each symbol $i \in G1$ is placed in each position and that each copy of each symbol $i \in G1$ is assigned to one and only one position of the sequence. It is assumed that each copy of symbols $i \notin G1$ are placed in the positions that are free of symbols $i \in G1$. Note that the order in which the symbols $i \notin G1$ are placed is irrelevant since they do not contribute to the objective function. Constraint (5) fixes the first copy of symbol $i*$ in the first position of the sequence in order to eliminate symmetric (equivalent) solutions (recall that a solution is defined by the distances between the consecutive copies of each symbol rather than by the absolute positions in which the copies are placed). Constraint (6) ensure the natural order between copies of the same symbol. Constraint (7) state the value of the objective function. Constraints (8) and (9) ensure together that the value of the objective function is one in the set V of its possible values. Variable mrt and constraints (8) and (9) are not