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Single-Source Capacitated Multi-Facility Weber Problem—An iterative two phase heuristic algorithm

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ABSTRACT

Multi-Facility Weber Problem (MFWP), also known as continuous location–allocation problem, entails determining the locations of a predefined number of facilities in a planar space and their related customer allocations. In this paper, we focus on a new variant of the problem known as Single-Source Capacitated MFWP (SSCMFWP). To tackle the problem efficiently and effectively, an iterative two-phase heuristic algorithm is put forward. At the phase I, we aim to determine proper locations for facilities, and during the phase II, assignment of customers to these facilities is pursued. As an alternative solution method, a simulated annealing (SA) algorithm is also proposed for carrying out the phase I. The proposed algorithms are validated on a comprehensive set of test instances taken from the literature. The proposed iterative two-phase algorithm produces superior results when assessed against the proposed SA algorithm as well as a general MINLP Solver known as BARON. The latter is applied to produce optimal solutions for small sized instances and generate upper bound for medium ones.

1. Introduction

Multi-Facility Weber Problem (MFWP) concerns with how to determine the locations of a predefined number of uncapacitated facilities in a planar space and how to allocate a given set of customers to them aiming to minimize the total transportation cost between facilities and their allocated customers.

Although MFWP can easily be understood, the problem is intractable from the computational point of view. In fact, non-convexity and non-differentiability of the objective function along with the existence of multiple local minima are the most challenging features of the problem [1]. Furthermore, Sherali and Nordai [2] demonstrate that the problem is NP-hard, even if all of demand points are to be located on a straight line. Since the facilities are uncapacitated, it can easily be proved that in an optimal solution of the MFWP, each customer is satisfied by its nearest facility.

The optimal solution of MFWP could be infeasible in practice as some facilities may end up being located at unusable locations such as a lake, a mountain, etc. Nevertheless, determining the location of oil drills in a sea or a desert is a practical application for the MFWP (e.g., Rosing [3]). Another important by-product of the result could be used to curb the number of potential sites so as to mitigate the required time and cost for data gathering that is necessary to solve a discrete case. From the practical and economical viewpoints, the collection of the data regarding all facilities in the discrete case is a heavy burden financially that is not usually taken into account in the location literature; see Manzour-al-Ajdad et al. [4]. Particularly, taking into account the single-source and capacitated form of the MFWP is appealing as facilities have usually restricted capacity in reality, and customers are also interested to have interaction with one single facility due to achieving the economies of scale; see [4].

Capacitated MFWP (CMFWP) is a more practical variant of the MFWP where facilities with limited capacities are taken into account. In this paper, we focus on Single-Source Capacitated MFWP (hereafter SSCMFWP) where each customer has to receive its total demand only from a single capacitated facility. Surprisingly, investigation of the literature reveals that the SSCMFWP suffers from the lack of enough attention in spite of its many practical applications in reality (e.g., [1,3]).

Below the necessary notations used in the problem formulation are introduced followed by the corresponding mathematical model:

Parameters

m	number of facilities
n	number of customers
$c_j=(a_j,b_j)$	coordinates of the jth customer
w_j	requirement (demand) of the jth customer
q_i	capacity of the ith facility

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Decision variables

$p_i=(x_i,y_i)$ coordinates of the ith facility
z_{ij} 1; if the jth customer is assigned to the ith facility, 0; otherwise

$$\min z=\sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{(x_i-a_j)^2+(y_i-b_j)^2}z_{ij} \quad (1)$$

$$\sum_{i=1}^{m} z_{ij}=1 \quad j=1,2,\ldots,n \quad (2)$$

$$\sum_{j=1}^{n} w_j z_{ij} \le q_i \quad i=1,2,\ldots,m \quad (3)$$

$$z_{ij}\in\{0,1\} \quad i=1,2,\ldots,m \quad j=1,2,\ldots,n \quad (4)$$

$$(x_i,y_i)\in\Re^2 \quad i=1,2,\ldots,m \quad (5)$$

The objective function is to minimize the total Euclidean distances between the customers and the facilities. Eq. (2) indicates that each customer must be fulfilled by only one facility, and Eq. (3) denotes that the capacity of facilities should not be violated. Finally, Eqs. (4) and (5) stipulate the binary and continuous nature of the allocation and location variables, respectively.

It is also worth noticing that the problem is in connection with some other well-known optimization problems and hence paying enough attention to this fact may give us the opportunity to exploit the features of those problems. In this regard, the problem reduces to the Generalized Assignment Problem (GAP) when the locations of facilities are determined, see Guignard [5] for more information about GAP. Furthermore, CMFWP can be derived from the SSCMFWP by relaxing the integrality constraints, i.e., considering the z_{ij} variables as continuous ones in the interval [0,1] instead of binary variables. Noteworthy, for a given set of allocations, the SSCMFWP is transformed to m pure location problems each of which can be separately solved exactly using the well-known method given by Weiszfeld [6].

The purpose of this study is twofold: (1) developing two heuristic algorithms to deal with the SSCMFWP and (2) validating the proposed algorithms on a comprehensive set of test instances taken from the literature that would also be worthy for future studies in this area.

The remainder of the paper is organized as follows. In Section 2, we present a review of the relevant literature. Sections 3 and 4 address the proposed two-phase algorithm and SA solution method, respectively. Computational results are provided in Section 5. Finally, Section 6 concludes this paper and provides some research avenue that we believe to be worth exploring in the future.

2. Literature review

MFWP has considerably been probed in the literature as a widely used optimization problem. Rosing [7] develops a branch and bound algorithm to solve the problem. Krau [8] uses a column generation approach mixed with global optimization and branch-and-bound to tackle the problem. Since the exact methods are not capable of dealing with large-sized instances in a reasonable computational time, heuristic methods appeared to be the best way forward. Cooper [9] proposes an iterative heuristic method known as the Alternate Location–Allocation (ALA) algorithm for solving the MFWP. The algorithm is efficient in terms of solution quality and computational effort. Hansen et al. [10] deal with the MFWP by solving a *p-median* problem to optimally while considering all fixed points as potential facility sites and then apply the ALA algorithm to find proper locations for the facilities. Gamal and Salhi [11] embed two procedures, namely the furthest distance rule and the forbidden points, into the ALA algorithm in order to generate more efficient initial solutions. Brimberg et al. [12] put forward a solution approach by proposing a combination of Variable Neighborhood Search (VNS) and the ALA algorithm. Gamal and Salhi [13] investigate a two-phase heuristic method known as a cellular heuristic, and Salhi and Gamal [14] propose a genetic algorithm (GA) to solve the problem. Taillard [15] puts forward a decomposition heuristic and partitions the problem into the smaller sub-problems which are then solved by a heuristic algorithm, namely candidate list search.

In connection with CMFWP, Sherali et al. [16] introduce an exact method to tackle the problem with rectangular distances. In the proposed method, a reformulation of the problem as a mixed integer bi-objective linear programming model is proposed. Al-Loughani [17] presents an exact method to deal with the problem with Euclidean distances where a branch-and-bound algorithm is presented that implicitly/partially enumerates the vertices of the feasible region of the transportation constraints. Cooper [18] develops a heuristic method for the CMFWP known as the Alternate Transportation–Location (ATL). Aras et al. [19] propose a mixed integer linear programming approximation of the problem and then develop three heuristic methods to handle the constructed problem with Euclidean, squared Euclidean and l_p distances. A perturbation-based heuristic method is also introduced by Zainuddin and Salhi [20]. This heuristic method provides superior results in comparison to the classical ATL when tested on large-sized instances (n=50–1060) given in [12]. More recently, Luis et al. [21] embed the ATL algorithm [18] into a Greedy Randomized Adaptive Search Procedure (GRASP) in order to reach robust solutions. They demonstrate the accuracy of their algorithm empirically via comparing it with the optimal solutions found for small sized instances and recent heuristics for large sized instances.

It is puzzling why the number of studies regarding SSCMFWP is very scarce in spite of being practical in reality. Gong et al. [22] propose an iterative method including location and allocation phases known as Hybrid Evolutionary Method (HEM) to solve the problem. In the location phase, a GA is exploited to generate the proper locations for the facilities. When the locations of facilities are fixed, the problem reduces to a GAP. Then, in the allocation phase, a Lagrangean Relaxation (LR) is used to solve the resulting GAP where the capacity constraints are dualized. They generate 16 random samples to test the proposed algorithm. In addition, Doong et al. [23] put forward a generalized variant of continuous location-allocation problem in which the number of facilities is not predefined and fixed cost of facilities is also taken into account. It is worth noting that the model considered in this paper is similar to that of Gong et al. [22]. Nevertheless, the method given in [22] has a deficiency that is briefly discussed here. By taking into account some relevant studies regarding GAP (e.g., [5,24,25]), it can be concluded that when devising a Lagrangean relaxation solution method, a procedure must be embedded to modify the results of LR with the aim of satisfying the set of relaxed constraints and reaching to a feasible solution. But, there is no discussion in [22] to justify how the set of relaxed constraints i.e., capacity constraint is satisfied. It also should be noted that the LR method sometimes is not capable of generating any feasible solution for the GAP, for example see Klincewicz and Luss [24]. Hence, presenting a method to guarantee feasibility is necessary. With regard to applying the LR method to solve the GAP, interested reader may also consult with [5].

3. Solution framework

In this section, an iterative two-phase solution procedure is proposed. In the phase I, named as the location phase, the ALA