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An adaptive penalty based covariance matrix adaptation–evolution strategy

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ABSTRACT

Although most of unconstrained optimization problems with moderate to high dimensions can be easily handled with Evolutionary Computation (EC) techniques, constraint optimization problems (COPs) with inequality and equality constraints are very hard to deal with. Despite the fact that only equality constraints can be used to eliminate a certain variable, both types of constraints implicitly enforce a relation between problem variables. Most conventional constraint handling methods in EC do not consider the correlations between problem variables imposed by the problem constraints. This paper relies on the idea that a proper genetic operator, which captures mentioned implicit correlations, can improve performance of evolutionary constrained optimization algorithms. With this in mind, we employ a $(\mu+\lambda)$-Evolution Strategy with a simplified variant of Covariance Matrix Adaptation based mutation operator along an adaptive weight adjustment scheme. The proposed algorithm is tested on two test sets. The outperformance of the algorithm is significant on the first benchmark when compared with five conventional methods. The results on the second test set show that algorithm is highly competitive when benchmarked with three state-of-art algorithms. The main drawback of the algorithm is its slightly lower speed of convergence for problems with high dimension and/or large search domain.

1. Introduction

Global optimization is an essential part of any engineering, economics and social system. Since Holland's [1] ground breaking work, global optimization approaches inspired by nature have been widely used. Among those algorithms, population based algorithms are known for their global search ability and very precise approximation of global solutions despite their relatively slow convergence for some problems and approximation bias [2].

When solving constrained optimization problems, traditional optimization techniques generally demonstrate poor global search performance for non-convex problems, as any local optimal point can satisfy their convergence condition [3].

Although most of unconstrained optimization problems with moderate to high dimensions can be easily handled with Evolutionary Algorithms (EA), constrained optimization problems (COPs) with inequality and equality constraints are very hard to deal with [4]. The difficulty level also depends on the dimension, number of inequality and equality constraints as well as structural specifications of the problem, including sparsity of the feasible domain, the position of the global solution (for instance, a solution lying on the boundary of feasible domain), non-separable character of the variables and nonlinear structure of the objective function. Thus, COPs require an exhaustive search of the feasible domain [4,5]. Despite the fact that there have been numerous constraint handling techniques proposed by researchers [6], there is still a need to design new methods which have to be computationally efficient and reliable [7]. In the design of new algorithms most researchers have focused to determine how to generate feasible individuals while maintaining a reasonable ratio between feasible and infeasible members in a population so that the algorithm is able to jump in a sparse feasible domain [8–10].

A COP may consist of many equality and inequality constraints. The equality type one imposes a strict relation between problem variables and can be exploited to determine any unknown in terms of other variables in the equation. However, sometimes the direct use of them may be impossible or computationally expensive if equality constraints are not analytically solvable. The imposed relation originated from an equality constraint is a *strong relation* as it narrows down the feasible space drastically.

While the inequality constraints do not allow a direct elimination of a problem variable, they also establish relations, in a less strict manner than equality constraints, between the unknowns. These relations are rather *weak relations*. The strong and weak relations between problem variables imposed by the constraints can be exploited in an indirect way even if they do not allow a direct exploitation. Most conventional constraint handling methods in Evolutionary Algorithms (EAs) overlook the following.

- Each inequality and equality constraint implicitly enforce a relation between variables.

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- This relation can be captured by a covariance matrix of the population and can be exploited to find the global optimum if it is incorporated to a proper constraint handling method.

This paper relies on the idea that the precision and convergence power of constrained optimization methods can be improved by taking the covariance matrix of the population into account, as it considers the strong and weak relations indirectly. By this way, the population may be allowed to move from a feasible region to another one along the relation indicated by the constraints. To obtain the desired behavior, the Covariance Matrix Adaptation (CMA) based mutation strategy suggested by Hansen and Ostermeier [11] is employed along with an adaptive penalty approach based on adaptive segregational constraint handling evolutionary algorithm (ASCHEA) [8].

The remainder of this paper is organized as follows: in Section 2, basic concepts and related works will be shortly discussed, while more attention will be paid to ASCHEA and CMA. Section 3 describes the proposed method in detail while Section 4 is devoted to a comprehensive parametric analysis of the proposed model. The results of the study will be demonstrated and discussed in Section 5. Lastly, Section 6 concludes the study and summarizes the findings.

2. Constrained optimization

2.1. Basic concepts

An n dimensional COP can be defined by two components: an objective function to be maximized or minimized, and several inequality and equality constraints. The general structure is defined as

$$\text{Minor max} f\left(\vec{x}\right), \vec{x} = [x_1, \ldots, x_n]^T \in F \subseteq S \subseteq \mathbb{R}^n$$

subject to

$$\psi_i(\vec{x}) \leq 0, \quad i = 1, \ldots, r$$

$$\phi_j(\vec{x}) = 0, \quad j = r+1, \ldots, m$$

where $S = \{\vec{x} \in \mathbb{R}^n | \boldsymbol{l} \leq \vec{x} \leq \boldsymbol{u}\}$ and $F = \{\vec{x} \in S | \psi_i(\vec{x}) \leq 0 \ and \ \phi_j\left(\vec{x}\right) = 0\}$, $\vec{x}$ is solution vector $\vec{x} = [x_1, \ldots, x_n]^T$, r is the number of inequality and m–r is the number of equality constraints. The equality constraints are usually converted into inequalities by adding a small tolerance $\varepsilon > 0$ where an equality constraint j is rephrased as $\left|\phi_j\left(\vec{x}\right)\right| - \varepsilon \leq 0$. The same approach will be used in the present work.

2.2. Related work

As mentioned before, various different techniques have been proposed to handle COPs. An extended survey can be found in [6,12,13]. The constrained optimization evolutionary algorithms (COEAs) can be classified in the following four categories illustrated in Fig. 1: feasibility maintenance, penalty function, separation of constraint violation and objective value, and multiobjective optimization evolutionary algorithms (MOEA) [12].

Approaches based on feasibility maintenance aim to bring the individuals to the feasible domain. Repairing infeasible individuals and homomorphous mapping are two methods that dominate this category. The repaired individuals are replaced or sometimes used only for evaluation purposes [14]. To repair infeasible individuals, problem specific operators must be designed, which may not be an efficient method in some cases and repair operator may introduce a strong bias in the search. This may harm the evolutionary process itself [15]. Homomorphous mapping tries to maintain the feasibility of population by mapping the feasible domain onto a hypercube and performing evolutionary operators within the hypercube. The offspring, guaranteed to be feasible, are then transferred back to the definition domain [16]. Despite its secure feasibility maintenance property, homomorphous mapping comes along with high computational cost because back and forward mapping must be conducted through some optimization methods for each individual [12,16].

The methods based on penalty functions are the most popular approaches, thanks to their simplicity and easy application [8]. They rely on penalizing the infeasible individuals, so that a feasible point will be superior to an infeasible point of comparable fitness. However, two main questions arise in penalty-based method:

- How to adjust the penalty weights for each constraint.
- How to maintain a certain percentage of infeasible individuals in the population, which allows determining the global optimum in highly sparse feasible space.

The penalty weights must be tuned very carefully in order to avoid the above mentioned two problems. A small penalty level

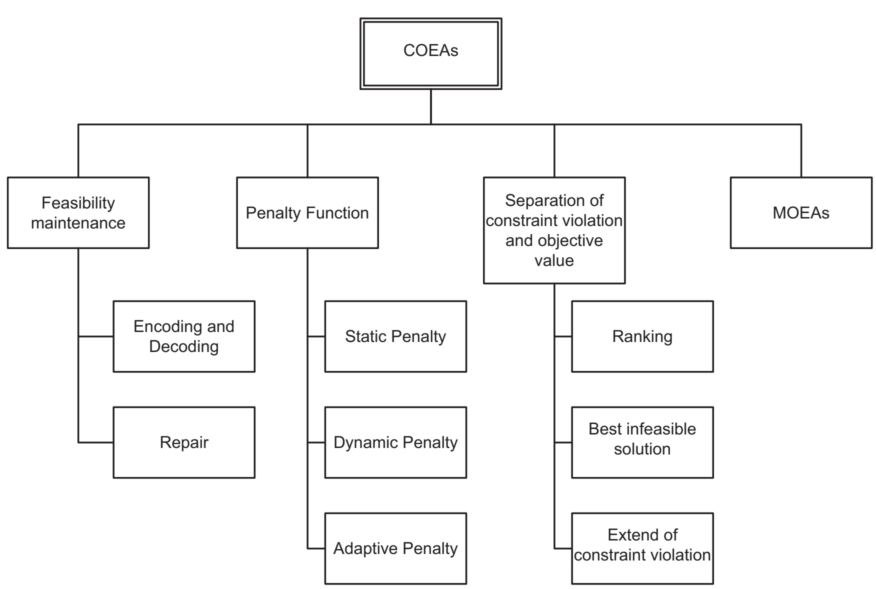

Fig. 1. The taxonomy of COEAs.