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Applying two efficient hybrid heuristics for hub location problem with fully interconnected backbone and access networks

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ABSTRACT

This paper considers the design of two-layered networks with fully interconnected backbone and access networks. The problem, a specific application of hub location to network design, is known as fully interconnected network design problem (FINDP). A novel mathematical programming formulation advantageous over an earlier formulation is presented to model the problem. Two hybrid heuristics are proposed to solve the problem, namely SA_{VNS} and TS_{VNS} which incorporate a variable neighborhood search (*VNS*) algorithm into the framework of simulated annealing (*SA*) and tabu search (*TS*). The proposed algorithms are able to easily obtain the optimal solutions for 24 small instances existing in the literature in addition to efficiently solve new generated medium and large instances. Results indicate that the proposed algorithms generate high quality solutions in a quite short CPU time.

1. Introduction

A communications network consists of a set of nodes that transmit messages or information, in the form of voice, data and/or video, over communication links (e.g., copper cables, fiber-optic cables, radio links, or satellite links). The areas that can be served by a communication network range from spaces as small as a single building, to large geographic areas, and even the globe itself. Such networks have a wide range of applications, including simple telephone calls, video teleconferences, packet data transmission, and distributed computer processing, among others.

It is a difficult problem to design an efficient yet cost-effective network, especially in the case of large scale networks with many nodes that share traffic. In some special cases, like wired communication networks, network configurations are usually organized in a hierarchical structure based on two layers or more. This structure has been proven robust to changing demands and upgrades and is seen as the right compromise between cost and redundancy [1].

This paper considers networks with two layers in which hub nodes can act as switching points for the communications traffic. A number of "access" networks connect the clusters' nodes to the hubs. A "backbone" network then interconnects these hubs. The connection between an access network and the backbone network is established using hub nodes. The traffic between two nodes belonging to different access networks has to be routed via the hubs in the backbone network. After passing through the backbone network, the traffic again uses an access network again to travel from a hub to its destination node. Fig. 1 depicts an example of networks with the described properties. In this figure, hubs are shown as squares and thin lines are used to connect nodes to each other within the access networks, while thick lines show connections between hub nodes throughout the backbone network. Meanwhile, the clusters with a hub node are marked with dashed lines, each representing an access network.

Designing such hierarchical or layered networks comprises a number of interrelated questions that have to be answered: which nodes should be selected as hubs, how the clusters should be defined, and which interconnections may be allowed. Since these problems are interrelated, they should be addressed by an integrated approach in order to ensure an optimal solution.

Following the approach in Thomadsen and Larsen [1], the joint selection of hubs and clustering of nodes in two-layered networks is considered. All interconnections in the backbone network and access networks are assumed to be allowed. As evident from Fig. 1, both the access networks and the backbone network are fully interconnected, which means that for any pair of nodes in the backbone network or in any of the access networks, there is a link connecting them to each other. In this problem, called fully interconnected network design problem (FINDP) in the literature, the total cost of direct links established to handle communications between pair of nodes in both the backbone and the access

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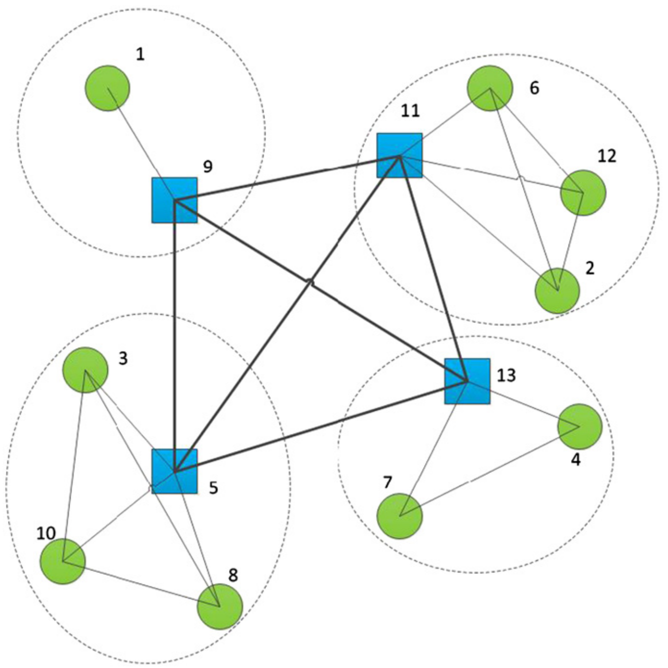

Fig. 1. An example of two-layered fully interconnected backbone and access networks.

networks is to be minimized. However, in this problem, the flow costs are not considered, in contrast to the conventional hub location problems. Note that the communication between two nodes in the different access networks must be routed via the hubs in the backbone network.

A review of two-layered network design problems was performed by Klincewicz [2]. A significant portion of the research work focuses on networks with fully interconnected backbone network and a star access network. The interested readers are referred to Ernst and Krishnamoorthy [3], Skorin-Kapov et al. [4] and Alumur and Kara [5] for more information about the design of these networks which is known as hub location problem in the literature.

So far, several heuristics have been developed for the hub location problem. O'Kelly [6] proposed two enumeration-based heuristic strategies with different assignment methods: HEUR1, HEUR2. In HEUR1, assignment of every node is done based on the nearest hub, while for the HEUR2 assignment is implemented based on the nearest or the second nearest hub. Exchange heuristics, including both the single and double exchange procedures were investigated in Klincewicz [7]. These heuristics are concluded to be superior to those proposed by O'Kelly [6]. In Klincewicz [8], the problem was solved using heuristics based on tabu search (*TS*) as well as a greedy randomized adaptive search procedure (GRASP).

Abdinnour-Helm and Venkataramanan [9] developed a branch-and-bound (B&B) algorithm and a genetic algorithm (*GA*) to solve the uncapacitated single allocation hub location problem (USAHLP). The exact B&B procedure was time consuming and limited to solve smaller size problems, while the *GA* can be efficiently and quickly used for large problems. A hybrid heuristic based on *GA* and *TS* is proposed for the USAHLP in Abdinnour-Helm [10]. In this hybrid heuristic, *GA* is utilized to determine number and location of the hubs, while *TS* is employed to find an optimal assignment of spokes to the hubs. Abdinnour-Helm [11] proposed a solution heuristic based on simulated annealing (*SA*) to solve the uncapacitated single allocation *p*-hub location problem (USApHMP). In their work, *SA* was applied to the assignment stage of the problem. Topcuoglu et al. [12] presented a *GA*-based method to determine number and location of hubs and also allocation of the non-hub nodes which led to good results in terms of both solution quality and CPU time as compared with those previously reported by Abdinnour-Helm [10]. Silva and Cunha [13] utilized both multi-start and two-stage tabu search based heuristics for USAHLP which yielded optimal or best-known solutions that are also comparable with the previous works in terms of CPU time. In Ilic et al. [14], a generalized variable neighborhood search (GVNS) heuristic was developed to solve USApHMP and some of the best known solutions were improved.

The FINDP is a fixed charge problem in which a cost is incurred for establishing links [1]. Crainic et al. [15] considered a capacitated version of the fixed charge network design problem (FNDP) where network arcs have finite capacities. They introduced Lagrangian-based bounding methods to solve the problem. Their approach attains very high quality lower bounds as compared with those obtained by linear programming relaxation of proposed mixed integer programming formulation. A comprehensive review of papers where Bender's decomposition algorithms are proposed to FINDP was presented by Costa [16]. Feresmans et al. [17] conducted another survey on papers considering two layered network design problems where it is assumed that the clusters are predetermined. These networks named generalized network design problems (GNDP) were formally defined and their properties as well as some applications were provided. Thomadsen and Stidsen [18] addressed such a problem, where the backbone is a fixed charge network design problem. They modeled the problem as mixed integer programming and then developed a branch-cut-and-price algorithm incorporating LP relaxation of MIP model in its implementation.

A typical application of FINDP could be found in setting up a computation cluster of computers. This can be done such that all computers get directly connected with each other which results in greater establishment costs with difficulty on its management. Another option is designing a fully interconnected network for these computers so that they are divided into two layers, a backbone network of hub computers that ensures the fast access and access networks of non-hub computers using higher technologies between hubs. First work on FINDP was performed by Thomadsen and Larsen [1] where a mathematical programming formulation was proposed for the problem. They also developed a formulation based on set partitioning model and column generation approach. Using this approach they were able to solve problems consisting of up to 25 nodes. Since the exact methods become prohibitively expensive to apply for larger instances, developing variants of algorithms to efficiently solve FINDP problem within an acceptable time is motivated. Recently, Ortiz-Garcia et al. [19] developed a parallel evolutionary algorithm to tackle this problem and demonstrated its good performance through a computational study. Although their proposed algorithm could reach smaller gaps for size of instances smaller than 15 nodes, it was still quite time consuming for large instances.

In this paper, a new quadratic programming model, based on standard hub location formulations, along with its linearized version is proposed which is comparable with the one developed by Thomadsen and Larsen [1] in terms of number of variables and constraints. Two hybrid heuristics are developed and compared for solving the FINDP problem. The first one uses *SA* combined with *VNS* referred to as SA_{VNS}, while the second one called TS_{VNS} uses *VNS* in the framework of *TS*. These hybrid heuristics exploit the advantages of above-mentioned individual metaheuristic components. The proposed algorithms are characterized by several aspects that enhance their effectiveness. First, a novel