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Service and capacity allocation in M/G/c/c state-dependent queueing networks

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Abstract

The problem of service and capacity allocation in state-dependent M/G/c/c queueing networks is analyzed and algorithms are developed to compute the optimal allocation c. The model is applied to the modeling of pedestrian circulation systems and basic series, merge, and split topologies are examined. Also of interest are applications to problems of evacuation planning in buildings. Computational experiments assert the algorithm's speed, robustness, and effectiveness. The results obtained indicate that the pattern of the optimal capacity surprisingly repeats over different topologies and it is also heavily dependent upon the arrival rate. Additional computational simulation results are provided to show the accuracy of the approach in all configurations tested.

Keywords: Queueing networks; Buffer allocation; Finite capacity; State dependent

1. Introduction

Many application problems including those in telecommunication, transportation, manufacturing, and service industries are most appropriately modeled as queueing networks with finite capacity and state-dependent service rates. Often, the finite capacities in the queues and state-dependent service rates further increase the complexity of solutions of these systems. In other cases, these assumptions may be relaxed. However, this paper is about applications for which it is fundamental to take into account finite capacities and state-dependent services for the sake of accuracy. Of particular interest are M/G/c/c state-dependent queueing networks, i.e., following Kendall's notation,

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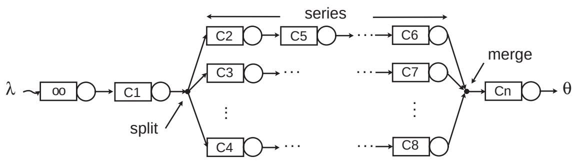

Fig. 1. A generic network topology.

queues with Markovian arrivals, general state-dependent service rates, c parallel servers, and the total capacity c, including the servers.

Fig. 1 illustrates a queueing network configured as a generic topology. Notice that the classical symbolic representation for each stochastic node is being used in this figure, as well as throughout this paper. However, it is worthwhile mentioning here that actually there is no queue at each M/G/c/c node since the number of parallel servers equals the total capacity.

The use of queueing theory for the analysis of congestion has a long and storied existence. In the past, queueing networks have been important tools in the study of traffic light synchronization, in the analysis of vehicles at intersections [1], and in the evaluation of traffic flow by using a simplistic deterministic approach [2]. Nowadays, probably due to the increasing speed and reduced costs of the modern computer systems, more sophisticated models have been developed. These models have been used in applications such as pedestrian/vehicular network traffic analysis [3–5], synthesis [6–9], and accumulating conveyor systems [10].

The main reason of this paper is to extend the development of algorithms for optimal service and capacity allocation in M/G/c/c state-dependent queueing networks, for a fixed generic network topology. In particular, the interest lies in pedestrian network applications, configured as a generic combination of basic series, merge, and split topologies as illustrated in Fig. 2, but the extensions to other networks with state-dependent service rates should be obvious.

The rest of this paper is organized as follows. Section 2 presents a mathematical programming formulation for the service and capacity allocation (SCA) problem. Section 3 presents the analytical stochastic model used to describe pedestrian flows and Section 4 describes the proposed algorithm. Computational experiments with the proposed algorithm are presented in Section 5. In order to illustrate the usefulness of the optimization algorithm in evacuation problems, Section 6 is dedicated to show results for a ten-story building evacuation network. Finally, Section 7 closes the paper with a summary and concluding remarks.

2. Problem statement

2.1. Notation

The notation used throughout the text is provided below:

c corridor capacity in number of occupants
$\mathbf{c}$ capacity vector $(c_1, c_2, \ldots)^{\mathrm{T}}$
f_i cost per unit of capacity
l corridor length in meters