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Scheduling two-machine no-wait open shops to minimize makespan

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Abstract

This paper examines the problem of scheduling two-machine no-wait open shops to minimize makespan. The problem is known to be strongly NP-hard. An exact algorithm, based on a branch-and-bound scheme, is developed to optimally solve medium-size problems. A number of dominance rules are proposed to improve the search efficiency of the branch-and-bound algorithm. An efficient two-phase heuristic algorithm is presented for solving large-size problems. Computational results show that the branch-and-bound algorithm can solve problems with up to 100 jobs within a reasonable amount of time. For large-size problems, the solution obtained by the heuristic algorithm has an average percentage deviation of 0.24% from a lower bound value.

Keywords: Scheduling; No-wait; Open shop; Branch-and-bound; Heuristic

1. Introduction

We consider the problem of minimizing makespan in a two-machine no-wait open shop environment. The problem can be stated as follows. There are n independent jobs to be processed through two machines M_1 and M_2. Each job i consists of two operations, O_{i1} and O_{i2}, with length a_i and b_i, respectively. The operations of a job can be performed in any order, but they must be performed without any interruption on machines and without any waiting in between machines. That is, each job must be processed continuously from its start to its completion (no-wait constraint). Hence, if necessary, the start of a job on one machine must be postponed so that the completion of this operation coincides with the beginning of the operation on the other machine. Furthermore, a machine can perform at most one operation at a time. The objective is to find a schedule that satisfies these

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constraints and minimizes the maximal job completion time of the n jobs. The problem is denoted as $O_2/nwt/C_{\max}$, following the notation by Graham et al. [1].

Scheduling problems with no-wait constraints occur in many industries. For instance, in hot metal rolling industries, where the heated metal has to undergo a series of operations at continuously high temperatures before it is cooled in order to prevent defects. Similarly, in the plastic molding and silverware production industries, a series of operations must be performed to immediately follow one another to prevent degradation. Other examples include chemical and pharmaceutical industries, food processing industries, and advanced manufacturing environments. The reader may refer to the papers by Hall and Sriskandarajah [2] and Goyal and Sriskandarajah [3] for a detailed discussion of the applications and research on no-wait scheduling problems.

Owing its practical applications and theoretical issues, no-wait scheduling has received an increasing amount of research attention in recent years. However, as shown in the survey paper by Hall and Sriskandarajah [2], much of the work on no-wait scheduling with makespan as the objective has been focused on flow shop problems. Gilmore and Gomory [4] propose an $O(n \log n)$ algorithm for the problem $F_2/nwt/C_{\max}$. Rock [5] proves that the problem $F_3/nwt/C_{\max}$ is strongly NP-hard. Piehler [6] shows the equivalence of the problem $F_3/nwt/C_{\max}$ to a traveling salesman problem. Many heuristics have been developed for the problem $F_m/nwt/C_{\max}$; see, for example, Reddi and Ramamoorthy [7], Wismer [8], Gangadharan and Rajendran [9], Rajendran [10], and Aldowaisan and Allahverdi [11]. For the case of open shop no-wait scheduling with makespan as the objective, most of the current research has been restricted to problems with unit processing time operations; see, for example, Coffman and Graham [12], Ullman [13], Gonzales [14], and Brucker et al. [15].

The problem $O_2/nwt/C_{\max}$ is shown to be strongly NP-hard by Sahni and Cho [16]. Based on the Gilmore and Gomory algorithm, Sidney and Sriskandarajah [17] present an heuristic algorithm for this problem, and establish a performance bound. Yao and Soewandi [18] develop a random search algorithm for the same problem, and provide computational experiments to demonstrate its performance. Yao and Lai [19] propose a genetic algorithm to solve this problem. We are not aware of any previous attempt to optimally solve no-wait open shop scheduling problems with general processing time operations. In this paper, we present both heuristic and exact algorithms for the problem $O_2/nwt/C_{\max}$. The proposed exact algorithm, based on a branch-and-bound scheme, solves the problem with up to 100 jobs within a reasonable amount of time, while the proposed heuristic quickly finds extremely high quality solutions for large-size problems.

The rest of this paper is organized as follows. In Section 2, a two-phase heuristic algorithm is proposed for solving large-size problems in practice. In Section 3, a branch-and-bound algorithm is developed to optimally solve the problem. Some dominance rules incorporated into the proposed branch-and-bound algorithm are given in Section 4. Computational results are provided in Section 5 followed by our conclusions in Section 6.

2. An heuristic algorithm

In this section, we present an efficient heuristic algorithm that produces a good solution quickly for the problem $O_2/nwt/C_{\max}$. Note that the makespan ($C_{\max}$) of a feasible schedule S is equal to the value $\max\{\sum_{i=1}^{n} a_i + I_1, \sum_{i=1}^{n} b_i + I_2\}$ where $I_1(I_2)$ is the total idle time on machine $M_1(M_2)$